Interaction between large and small scales in the canopy sublayer

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[1] Two characteristics that distinguish canopy sublayer (*CSL*) turbulence from its atmospheric surface layer (*ASL*) counterpart are short-circuiting of the energy cascade and formation of Kelvin-Helmholtz (*KH*) vortices near the canopy top. These two phenomena lead to nonlinear and poorly understood interactions between small and large scale eddies within the *CSL* absent from classical *ASL* turbulence. Using velocity scaling arguments and nonlinear time series analysis, we explore the degree of interaction between large and small scales in a canopy composed of densely arrayed cylinders. We found that such interactions are dynamically divided into four regions depending on the distance from the wall, and posses various degrees of nonlinearity and interaction strengths. The broader impact to *CSL* Large Eddy Simulations (*LES*) and low-dimensional dynamical systems (*LDDS*) models of coherent eddies is briefly discussed. *INDEX TERMS:* 3379 Meteorology and Atmospheric Dynamics: Turbulence; 0315 Atmospheric Composition and Structure: Biosphere/atmosphere interactions; 1869 Hydrology: Stochastic processes; 1894 Hydrology: Instruments and techniques. **Citation:** Poggi, D., A. Porporato, L. Ridolfi, J. D. Albertson, and G. G. Katul (2004), Interaction between large and small scales in the canopy sublayer, *Geophys. Res. Lett.*, *31*, L05102, doi:10.1029/2003GL018611.

1. Introduction

[2] The importance of quantifying the magnitude and the degree of nonlinearity of small and large scale eddy interactions within the *CSL* is a logical first step for developing subgrid models (*SGM*) for canopy *LES*, constructing closure models for the dissipation rate (ϵ) budget of turbulent kinetic energy (*TKE*), or deriving LDDS representation of coherent eddies for scalar transport within the *CSL*. Much of our understanding of small-scale turbulence is shaped by Kolmogorov's theory for the inertial subrange (*ISR*). An *ISR* forms when *TKE* cascades, on average, from lower to higher wavenumbers (K_w) at a rate identical to ϵ and independent of K_w. Eddies populating the *ISR* are sufficiently distant (in K_w domain) from the anisotropic energy-containing eddies and have been through sufficiently many non-linear interactions (vortex stretching) so they have lost any original anisotropy imposed by large scales. Because of the wakes produced by canopy elements, direct interaction between large and small scales (e.g., shortcircuiting) within the *CSL* occurs leading to difficulties in parameterizing the statistical properties of small scale eddies or energy flow to and from large eddies. Classical time/frequency analysis is commonly used to identify the statistical properties of energetic eddies or some local or global scaling properties of fine scale eddies [*Finnigan*, 2000; *Katul et al.*, 2001]. It is not evident whether such techniques can quantify the scale-wise interactions should this interaction be dominated by strongly nonlinear dynamics.

[3] A combination of simplified velocity scaling arguments and techniques from nonlinear time series analysis are used to explore the magnitude and the degree of nonlinearity of the interaction between large and small scales within and just above a laboratory model canopy. Our primary objective is to investigate whether the *CSL* can be divided into layers or regions that possess dynamically similar types of nonlinearity and/or interactions between large and small scales. The experimental setup comprises of high Reynolds number (R_e) flow through densely arrayed cylinders. This elementary configuration has the added benefit in that wake production occurs at a known length scale [*Poggi et al.*, 2004a]. The data from this experiment are analyzed in two ways. The first quantifies the magnitude of the interaction between small and large scales with respect to scale and distance from the wall (z) using the mutual information content (*MIC*). The second quantifies the degree of nonlinearity by contrasting the *MIC* with two linear methods. These methods include a linearized *MIC* applied to the measured time series [*Paluš*, 1995], and the *MIC* applied to surrogate time series [*Theiler et al.*, 1992; *Schreiber and Schmitz*, 2000].

2. Experiment

[4] The experimental setup is described in the work of *Poggi et al.* [2004a, 2004b]; however, a brief review is provided. The experiment was conducted in a re-circulating rectangular flume 18 m long, 0.90 m wide, and 1 m deep. The canopy is composed of vertical cylinders, 12 cm high (h) and 4 mm in diameter (d_r) with a frontal area index (λ_{FAI}) of 1072 rods m^{-2}. This λ_{FAI} results in a drag coefficient (C_d) comparable to those reported for densely forested ecosystems. The longitudinal velocity time series $u(t)$ was measured by a 2-*D* Laser Doppler Anemometry. With such λ_{FAI}, dispersive fluxes are small and a single measurement sufficiently represents the planar statistics [*Poggi et al.*, 2004c] at a given z. We sampled the *CSL* and the *ASL* at 1 cm vertical increments via 30 runs. The sampling duration and frequency per run were 3600 s and 2500–3000 Hz, respectively.

3. Nonlinear Analysis

[5] The analysis rests on three assumptions: 1) if small and large scales interact, then the small scale energy must

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contain "information" injected from larger scales, 2) the characteristic local energy of small scales at time t is $|\Delta u_\tau(t, z)|^2 = |u(t + \tau, z) - u(t, z)|^2$ where τ is the time lag. The characteristic energy for the large scales is approximated by $u'(t, z)^2 = [u(t, z) - U(z)]^2$ [e.g., *Praskovsky et al.*, 1993], where $U(z)$ is the time averaged velocity, 3) the interaction between small and large scales can be quantified using the *MIC* based on the standard Shannon entropy [*Shannon*, 1948]. For simplicity, let Δu^2 and u^2 be the small and large scale energy series, respectively. The entropy of the distribution for each variable and the joint entropy between Δu^2 and u^2 can be expressed as

$$H^{(\Delta u^2)} = -\sum_i p_i^{(\Delta u^2)} \ln p_i^{(\Delta u^2)} \tag{1}$$

$$H^{(\Delta u^2, u^2)} = -\sum_{i,j} p_{i,j}^{(\Delta u^2, u^2)} \ln p_{i,j}^{(\Delta u^2, u^2)}, \tag{2}$$

where $p_i^{(\Delta u^2)}$ and $p_j^{(u^2)}$ are the probability distribution (pdf) of Δu^2 and u^2 respectively, and $p_{i,j}^{(\Delta u^2, u^2)}$ is the joint probability distribution. The subscripts denote partitioning of probabilities $p_1, \ldots, p_M$, with $p_1 + \ldots + p_M = 1$. The "energetic" information content exchanged between large and small scales is quantified by $I_{r,z}^{(\Delta u^2, u^2)}$, given by

$$\begin{aligned} I_{r,z}^{(\Delta u^2, u^2)} &= H^{(\Delta u^2)} + H^{(u^2)} - H^{(\Delta u^2, u^2)} \\ &= \sum_{i,j} p_{i,j}^{(\Delta u^2, u^2)} \ln \frac{p_{i,j}^{(\Delta u^2, u^2)}}{p_i^{(\Delta u^2)} p_j^{(u^2)}}. \end{aligned} \tag{3}$$

[6] The suffixes, chosen for notational simplicity, underline the dependence of the *MIC* on separation distances (r) and z. The r is computed from τ using the frozen turbulence hypothesis.

4. Linear Analysis

[7] The usefulness of the above non-linear analysis for systems that are not clearly deterministic is controversial [*Procaccia*, 1988]. Pure determinism in natural systems, if it exists, is unlikely to be discovered by such analysis because of the large degrees of freedom, the interaction with other surrounding systems, transient forcing terms or boundary conditions, and unavoidable measurement noise. It is also recognized that when many weakly coupled degrees of freedom are "active" and associated with high noise levels, their product is an approximate Gaussian process. If this is true, then the use of non-linear analysis (vis-a-vis linear analysis) is not necessary to assess the degree of interaction between large and small scales. We take advantage of this point to quantify the degree of nonlinearity existing in such interactions using two methods: 1) a linear version of the *MIC* proposed by *Paluš* [1995] and 2) the surrogate data analysis introduced by *Theiler et al.* [1992]. We chose these two linearity measures (defined next) because they are sensitive to different assumptions; hence, qualitative agreement between them adds confidence that the variation of the degree of nonlinearity with z is not an artifact of the method to quantify linearity. Briefly, the linear *MIC* is derived by assuming linear gaussian dependence and may not capture the entire linear characteristics of the signal. On the other hand, the surrogate data is constructed to preserve several statistical properties of the real signal but is amenable to producing synthetic nonlinearities [*Paluš*, 1995]. Hence, by analyzing how the degree of non-linearity varies with z via these two measures (described next) permit us to assess whether the *CSL* possess regions that are dynamically similar.

4.1. Linear Mutual Information

[8] Using (3) and supposing that the random variables Δu^2 and u^2 are normally distributed with zero mean and covariance matrix C, the linear *MIC* can be computed from

$$L_{r,z}^{(\Delta u^2, u^2)} = \frac{1}{2}\left[\sum_{i=1}^{2} \ln(c_{ii}) - \sum_{i=1}^{2} \ln(\sigma_i)\right] \tag{4}$$

where c_{ii} and σ_i are the diagonal and eigenvalues of C [*Paluš*, 1995]. The difference between $L_{r,z}^{(\Delta u^2, u^2)}$ and $I_{r,z}^{(\Delta u^2, u^2)}$ can be used to quantify the degree of nonlinearity, given by

$$\alpha_l = \frac{I_{r,z}^{(\Delta u^2, u^2)} - L_{r,z}^{(\Delta u^2, u^2)}}{I_{r,z}^{(\Delta u^2, u^2)}}. \tag{5}$$

4.2. Surrogate Time Series

[9] Generation of surrogate time series is not unique and several methods have been proposed including wavelet-based generation, phase randomization, and iterative schemes that preserve various moments and spectral properties. Here, we employ a widely used and general method in which surrogate time series are constructed from direct Monte Carlo simulations to preserve a priori specified linear statistics of the original time series. The choice of these statistics is subjective and their optimum choice remains an open problem [*Theiler et al.*, 1992; *Schreiber and Schmitz*, 2000]. We construct surrogate time series, u_s, that preserve the spectra and pdf of u using the procedure by *Theiler et al.* [1992]. For a finite time series, because the pdf and the frequency spectrum cannot be simultaneously satisfied, an iterative scheme proposed by *Schreiber and Schmitz* [1996] is adopted to preserve precisely the spectrum and approximately the pdf. Upon comparing the u and u_s pdfs, only minor differences were found. The spectra and pdf are logical choices because they are commonly reported in *CSL* field experiments. To increase statistical robustness, an ensemble of ten u_s was generated for each u run. The *MIC* was computed for each u_s run and surrogate ensemble *MIC* was computed by averaging. Another measure of nonlinearity, α_s, can be defined from the difference between the *MIC* for u and u_s using

$$\alpha_s = \frac{I_{r,z}^{(\Delta u^2, u^2)} - I_{r,z}^{(\Delta u_s^2, u_s^2)}}{I_{r,z}^{(\Delta u^2, u^2)}}. \tag{6}$$

[10] By preserving the u pdf, α_s may capture some nonlinear information content already present in the

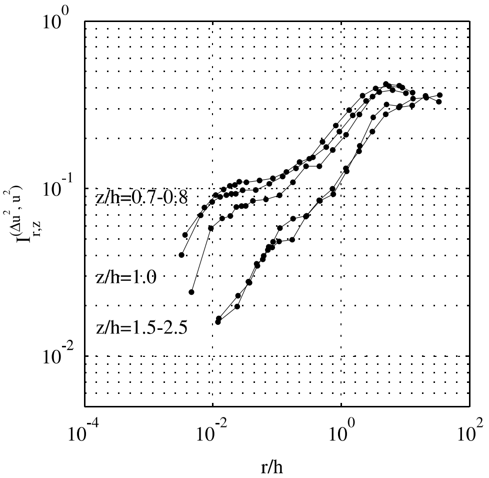

Figure 1. Variation of $I_{r,z}^{(\Delta u^2,u^2)}$ with separation distance (r/h) and depth (z/h).

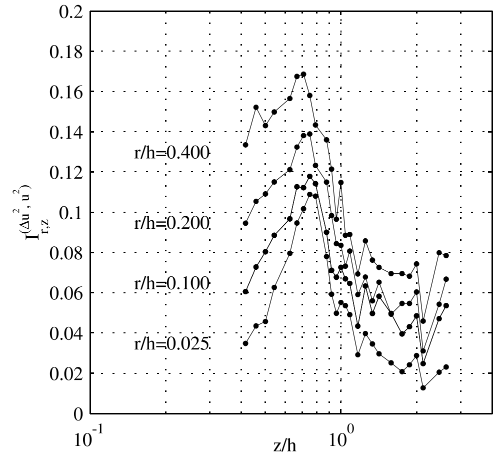

Figure 2. Variation of $I_{r,z}^{(\Delta u^2,u^2)}$ with normalized depth (z/h) and representative scales (r/h) derived from Figure 1.

moments of u. For this reason α_s and α_l are likely to differ in absolute value. However, to address our objective, what we seek is whether α_s and α_l profiles are qualitatively similar within the *CSL*. Again, using both measures has the added benefit of assessing whether the *MIC* is driven by interactions introduced from the dynamics or by differences in pdf's.

5. Results

[11] The results are discussed by contrasting $I_{r,z}^{(\Delta u^2,u^2)}$ computed in the *CSL* and *ASL* across different spatial scales (r) and for different z/h, respectively. Decreasing r means the interaction is evaluated at smaller scales. The $I_{r,z}^{(\Delta u^2,u^2)}$ for all (z/h) are shown in Figure 1. At the largest r/h and for all z/h, the interactions, as expected, are highest. At scales between r/h = 2/0.2 a region of power-law scaling with r emerges with an exponent insensitive to z. The most interesting part of $I_{r,z}^{(\Delta u^2,u^2)}$ is the region where $r/h \leq 0.2$. Here, $I_{r,z}^{(\Delta u^2,u^2)}$ is markedly different inside and outside the canopy. In particular, the interactions between small and large r appear much stronger inside the canopy, and the power-law scaling ceases to exist. In this region, the interaction between large and small scales appears "most enhanced" over its expected value (i.e., computed by extrapolating the power-law) and appears independent of (z/h) for within canopy flows. In Figure 2, the variation of $I_{r,z}^{(\Delta u^2,u^2)}$ with z/h for representative r/h portrays a clearer description of such interaction profiles. The *MIC* between large and small scales, when scales smaller than r/h = 0.3/0.4 are considered, peaks at (2/3 h). This behavior is likely linked with short-circuiting of energy due to canopy elements. Our analysis evidences the important role of short-circuiting in the exchange of energy between large and small scales and provides an experimental tool to evaluate the region (r, z) where this phenomenon appears significant. Near the wall and just above the canopy, the interaction is less pronounced. This behavior is perhaps expected above the canopy (i.e., *ASL*) where a Kolmogorov-like transfer of energy between scales occurs (i.e., weak direct interaction between large and small scales); however, in the region close to the wall, this behavior in $I_{r,z}^{(\Delta u^2,u^2)}$ was difficult to a priori guess. One plausible explanation is that the vorticity produced from the wall diminishes the impact of short-circuiting produced by the wakes. In Figure 3 the profiles α_l and α_s are presented for the same sections in Figure 2. As expected, the absolute value of α_l and α_s is not the same for all z/h and only agree with each other in the regions where the flow statistics are near Gaussian ($z/h = 0.9 - 2$). Despite the absolute differences in magnitude, the variation of α_l and α_s with z/h are quite similar. This similarity means that the scale-wise energy interactions at different z/h appears to be captured by both measures. Inside the canopy, the interaction between small and large scales can be reasonably described using a linear approach (though non-linearity is present). This small α_l suggests that the strong interaction between small and large scales shown in Figure 2 is governed by quasi-linear dynamics, perhaps attributed to the lower R_e. An opposite result is evident in the region just above the canopy in which the linear approach is not capable of describing the *MIC* between small and large scales. Although the interaction in this region is not as strong as that inside the canopy (Figure 2), a higher degree of non-linearity in the dynamics of such interaction appears to be present. This nonlinearity in the dynamics may well be

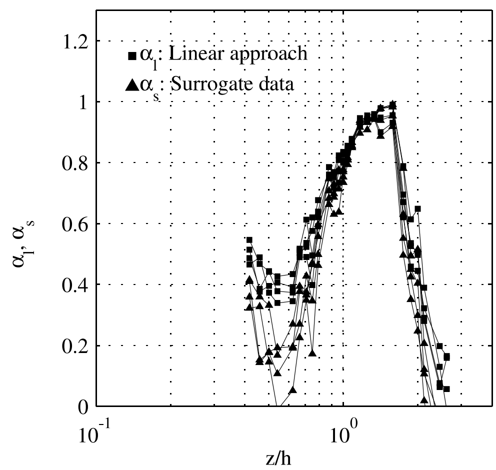

Figure 3. The variation of the two measures of non-linearity with depth (z/h) for several scales (r/h). For u_s, $L_{r,z}^{(\Delta u^2,u^2)} \simeq I_{r,z}^{(\Delta u^2,u^2)}$.

attributed to the nonlinear spectral dynamics of *KH* vorticity [*Bernal and Roshko*, 1986] and possible oscillations between attached eddies and *KH* [*Poggi et al.*, 2004a]. Finally, in the region well above the canopy (i.e., in the *ASL*) the degree of non-linearity in *MIC* dramatically decreases compared to its *CSL* counterpart. Such behavior may be attributed to a weak form of chaotic determinism governing these interactions in the *ASL*. Indirect support for this hypothesis is provided by numerous analytical and theoretical results on refinements to the Kolmogorov theory [see *Katul et al.*, 2001 for review]. However, this analysis alone does not prove that a weak form of chaotic determinism is governing the interactions in the *ASL* because of potential presence of stochastic noise [*Diks et al.*, 1995]. This point motivates future studies that may utilize several methodologies already developed for detecting determinism in several physics problems [e.g., *Bhattacharya and Kanjilal*, 1999; *Tsonis*, 2001]. Nonetheless, preliminary computations were performed to assess the capability of the proposed method in evaluating non-linearities in time series. We used the well-known Henon map in the chaotic regime to generate coupled nonlinear time series. Our choice of the Henon map stems from the fact that the Henon map possesses second order nonlinearity which resembles the advective terms in the Navier-Stokes equations (e.g., *Frisch*, 1995). A constrained randomization of the time series, a given randomization level, proposed by *Schreiber* [1998] was then adopted to preserve the PDF through a cost function involving all the higher order autocorrelations. The mutual information, the linear mutual information, and the degree of nonlinearity between the two series were computed and compared. We found that when the degree of non-linearity is large, both measures are near unity. Furthermore, both measures dramatically decrease when the degree of non-linearity decreases. This behavior confirms the capability of the proposed method to detect nonlinearity between two time series, at least qualitatively.

6. Conclusions

[12] We demonstrated that the degree of interaction between large and small scales in the *CSL* can be divided into four classes: 1) Deep inside the canopy ($z/h < 0.5$) for which the interaction appears weak and quasi-linear, 2) within the upper layers of the canopy ($0.5 < z/h < 1.0$), the interaction is strong but weakly nonlinear, 3) near the canopy top ($1.0 < z/h < 1.5$) the interaction is weaker when compared to its within canopy counterpart but strongly nonlinear and 4) well above the canopy ($z/h > 1.5$) in which the interaction is shown to be weak and linear in a stochastic sense. The broader impact of this work is that for *LES* models, the standard Smagorinski *SGM* approach must be revised to account for such non-local energy transfer between small and large scales. The degree of nonlinearity measured in this experiment can provide a necessary benchmark and framework for constructing more realistic *SGM* for *CSL* flows. Also, any attempt to model organized motion through a *LDDS* approach must consider the differences regarding small-large scale interactions within these four sublayers.

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