REPLY TO THE COMMENT BY BINK AND MEESTERS

Correspondence

GABRIEL KATUL and CHENG-I HSIEH
School of the Environment, Duke University, Durhain, North Carolina, U.S.A.

(Received in final form 17 April, 1997)

1. Introduction

Bink and Meesters (1997) (hereafter referred to as BM97) demonstrated that the relationship between R_{wT}, R_{wq}, and R_{Tq}

$$R_{Tq} = \frac{R_{wT}}{R_{wq}}, \tag{1}$$

proposed by Katul et al. (1995) (hereafter referred to as K95) can be formulated from simplifications to the covariance matrix determinant. Given that the determinant of the covariance matrix is positive semi-definite (see Jenkins and Watts, 1968, 88–89), BM97 showed that the inequality

$$\left| \frac{R_{xy}}{R_{xz}} - R_{yz} \right| \leq \frac{1}{|R_{xz}|} \sqrt{1 - R_{xz}^2} \sqrt{1 - R_{yz}^2}, \tag{2}$$

must be satisfied, where R_{ab} is the correlation coefficient between any two turbulent flow variables, and w, T, and q are the vertical velocity, air temperature, and specific humidity turbulent perturbations respectively. In (2) (Equation (17) in BM97), any combination of w, T, and q can replace x, y and z. BM97 used Equation (2) to investigate under what conditions Equation (1) is valid and whether R_{Tq} can be used as a surrogate for the relative efficiency ratio R_{wT}/R_{wq}. They argued that if the right-hand-side of (2) is small, then (2) reduces to

$$\frac{R_{xy}}{R_{xz}} - R_{yz} \approx 0. \tag{3}$$

Based on the above argument, BM97 concluded that

$$\frac{R_{wT}}{R_{wq}} \approx \begin{pmatrix} R_{Tq} & \text{if} & R_{wT} < R_{wq} \\ R_{Tq}^{-1} & \text{if} & R_{wT} > R_{wq} \end{pmatrix}, \tag{4}$$

which is more general than Equation (1) and does not suffer from the limitations and assumptions invoked by K95. Using high frequency w, T, and q measurements from

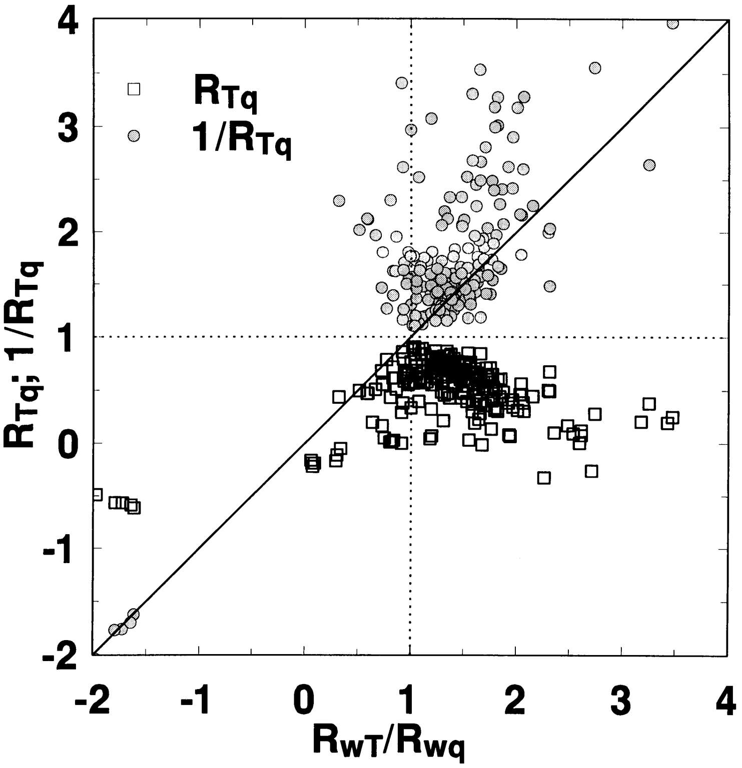

Figure 1. Variation of R_{Tq} (or $1/R_{Tq}$) with R_{wT}/R_{wq} using measurements from all three sites and conditions in K95.

the La Crau site (De Bruin et al., 1991; Kroon and Bink, 1996), BM97 concluded that predictions from Equation (4) are in good agreement with measurements. We investigated Equation (4) in Figure 1 using all measurements from K95 and found similar agreement to BM97 but the scatter in the K95 data is larger than that reported by BM97's Figure 1.

2. Comment on the Determinant of the Covariance Matrix and the K95 Conclusion

When BM97 compared the right-hand side (RHS) of Equation (2) with the left-hand side (LHS), they found that the RHS was an order of magnitude larger than the LHS for the La Crau data. We also carried out a similar comparison between the LHS and RHS using measurements from K95 and found that the RHS is about three times larger than the LHS as evidenced in Figure 2. Such an order of magnitude comparison between the LHS and RHS led BM97 to conclude that, while they agree with the K95 conclusion regarding the inadequacy of R_{Tq} to estimate the relative efficiency ratio R_{wT}/R_{wq}, they disagree with K95 that non-linearity in the expectation of T given w ($= E(T|w)$) is the primary reason. Here E is the conditional expectation operator (Hogg and Craig, 1978). Figures 3a and 3b demonstrate that such non-linearities exist for a wide range of atmospheric stability and surface roughness conditions thus invalidating Equations (7)–(10) in K95 or Equations (4)–(9) in BM97. Nevertheless, we do agree with BM97 that the inequality in Equation (2) holds irrespective of the functional relationship between $E(T|w)$ and w since the covariance matrix is always positive semi-definite. Since the RHS of Equation (2) is not small, as noted by BM97 and evidenced by the K95 measurements, the results in Equation (4) cannot be derived from Equation (2) (this point was also noted in the conclusion of BM97). In fact, a small RHS is attained if R^2_{xz} and/or R^2_{yz} are near unity. Since x, y, and z represent arbitrary random variables (and thus can be interchanged), it follows that all squared correlations must approach near unity, which again is unrealistic. Hence, the agreement between measurements and predictions by Equation (4) is due to a 'sufficiently' small LHS that cannot be deduced from the RHS. We agree with the main criticism of BM97 regarding the derivation of Equation (1); however, given the above order of magnitude analysis between the LHS and RHS, the determinant of the covariance matrix proposed by BM97 neither supports nor negates the conclusions in both K95 and BM97. Further theoretical and experimental research is needed to validate Equation (4).

3. Does the R_{Tq} Correction Improve the Scalar Flux Predictions?

For the flux-variance method (FVM), a Bowen ratio, defined as

$$\frac{\langle wT\rangle}{\langle wq\rangle} = \left(\frac{R_{wT}}{R_{wq}}\right)\frac{\sigma_T}{\sigma_q}, \tag{5}$$

is used to estimate latent heat fluxes ($\langle wq\rangle$) from sensible heat fluxes ($\langle wT\rangle$) and measured σ_q and σ_T, where $\langle\ \rangle$ is time averaging and σ_T and σ_q are the standard deviations of temperature and specific humidity respectively. For operational flux calculations, R_{wT}/R_{wq} is assumed unity. Since this assumption was the main motivation for deriving Equation (12) in K95, we re-evaluated the FVM-predicted latent heat fluxes using three approaches. All three approaches use the FVM-estimated sensible heat flux from measured σ_T as described in K95: approach 1 uses Equation (5) with measured σ_q and assumes $R_{wT}/R_{wq} = 1$; approach 2 uses Equation (5) with measured σ_q and determines R_{wT}/R_{wq} from measured R_{Tq} and Equation (4); and approach 3 uses the FVM formulation as in

$$\frac{\sigma_q}{q_*} = C_q\left(-\frac{z}{L}\right)^{-1/3}, \tag{6}$$

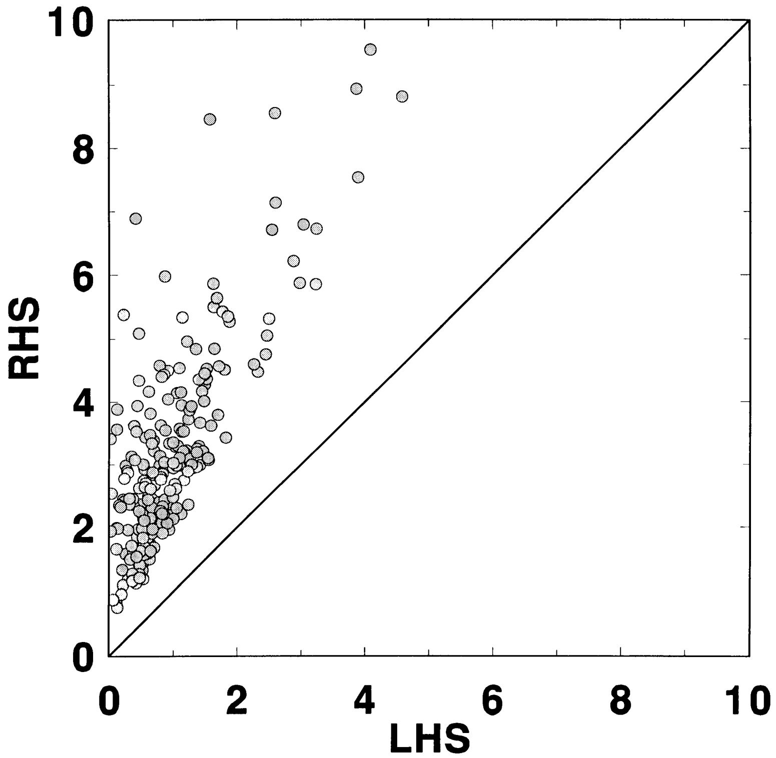

Figure 2. Assessment of the BM97 assumption resulting in Equation (4). The LHS and RHS are the left- and right-hand sides of Equation (2). The solid line is for LHS = RHS.

Table I

Comparison between measured ($F_{ec} = \langle wq \rangle$) and predicted ($F_p$) latent heat flux for all three sites in K95 (number of points n = 248) for $|R_{wT}/R_{wq}| > 0.1$ and $|R_{Tq}| > 0.05$. The regression model $F_{ec} = AF_p + B$ is used to compare all three approaches. The coefficient of determination (R^2) and the standard error of estimate (SEE) are also shown

Model	R^2	SEE (W m^{-2})	Slope (A)	Intercept (B)
Approach 1:				
($R_{wT}/R_{wq} = 1$)	0.51	49	0.56	18
Approach 2:				
$R_{wT}/R_{wq} = R_{Tq}$ or R_{Tq}^{-1} (n = 248)	0.35	57	0.48	59
Only for runs when $\lvert R_{wT}/R_{wq}\rvert > 1$ and $\lvert R_{Tq}\rvert > 0.05$ (n = 222)	0.66	42	0.93	27
Approach 3.1:				
($C_q = C_T = 0.95$)	0.51	49	0.57	18
Approach 3.2:				
($C_q = 1.3$)	0.51	49	0.77	18

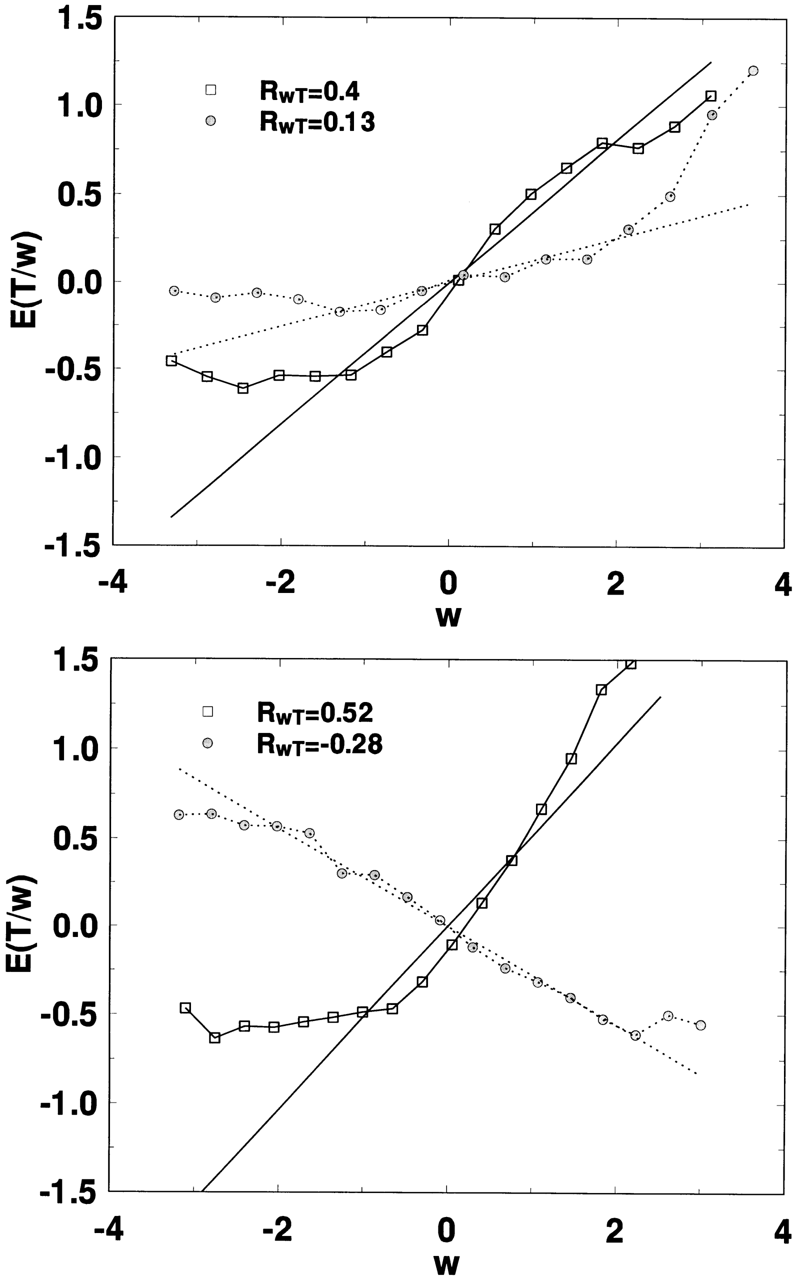

Figure 3. (a – upper) Estimated $E(T \mid w)$ from measurements at the Duke Forest grass-covered forest clearing for near-neutral $(R_{wT} = 0.13)$ and unstable $(R_{wT} = 0.40)$ atmospheric conditions. The solid line is from a linear expectation model. (b – lower) Same as Figure 3a, but for the Owens lake data set in Chu et al. (1996). For unstable and stable runs, $R_{wT} = 0.52$ and -0.23 respectively.

where $q_* = \langle wq \rangle / u_*$, u_* is the friction velocity, L is the Obukhov length, and z is the height above the zero-plane displacement (see Katul et al., 1995). In Equation (6), the latent heat flux can be written as

$$\langle wq \rangle = \frac{\sigma_q}{C_q} \left(\frac{kgz}{\langle T_a \rangle} \langle wT \rangle \right)^{1/3}, \tag{7}$$

where $\langle T_a \rangle$ is the mean air temperature, and C_T and C_q are similarity constants for heat and water vapour respectively. For the measurements in K95, C_T = 0.95 and C_q = 1.30 (see Figures 2b and 2c in Katul et al., 1995). In approach 3, $\langle wq \rangle$ is estimated with C_q = C_T = 0.95 (approach 3.1) and C_q =1.3 (approach 3.2). We note that several studies suggested that $C_T = C_q$ over uniform surfaces (Ohtaki, 1985) but that $C_T < C_q$ for non-uniform surfaces (see e.g. Roth and Oke, 1995; Hogstrom and Smedman-Hogstrom, 1974; Katul et al., 1995). A summary of these comparisons is shown in Table I for $|R_{wT}/R_{wq}| > 0.1$ and $|R_{Tq}| > 0.05$ (n = 248 runs). The conditions $|R_{wT}/R_{wq}| > 0.1$ and $|R_{Tq}| > 0.05$ reduce the total runs from n = 274 (as in the original K95 experiments) to n = 248 and are necessary to avoid unrealistic predictions of $\langle wq \rangle$. Approach 2 best reproduces the eddy-correlation $\langle wq \rangle$ measurements when $|R_{wT}/R_{wq}| > 1$ (and $|R_{Tq}| > 0.05$) as evidenced by the regression results of Table I. If all the data points are used (n = 248), then no clear advantage to using the BM97 correction for R_{wT}/R_{wq}, vis-à-vis assuming $R_{wT}/R_{wq} = 1.0$, is evident. We thank Bink and Meesters (1997) for their critical assessment and comments on Equation (1).

References

Bink, N. J. and Meesters, A. G. C. A.: 1997, 'Comment on Estimating Surface Heat and Momentum Fluxes Using the Flux-Variance Method above Uniform and Non-Uniform Terrain', *Boundary-Layer Meteorol.* **84**, 497–502.

Chu, C. R., Parlange, M. B., Katul, G. G., and Albertson, J. D.: 1996, 'Probability Density Functions of Turbulent Velocity and Temperature in the Atmospheric Surface Layer', *Water Resour. Res.* **32**, 1681–1688.

Courant, R. and Hilbert, D.: 1989, *Methods of Mathematical Physics*, Vol. I, John Wiley and Sons, 561 pp.

De Bruin, H. A. R., Bink, N. J., and Kroon, L. J. M.: 1991, 'Fluxes in the Surface Layer under Advective Conditions', in W. Schumgge and J. C. Andre (eds.), *Land Surface Evaporation*, Springer-Verlag, pp. 157–169.

Hogg, R. V. and Graig, A. T.: 1978, *Introduction to Mathematical Statistics*, Fourth Edition, Macmillan Publishing, 438 pp.

Hogstrom, U. and Smedman-Hogstrom, A. S.: 1974, 'Turbulence Mechanisms at an Agricultural Site', *Boundary-Layer Meteorol.* **7**, 373–389.

Jenkins, M. G. and Watts, D. G.: 1968, *Spectral Analysis and its Applications*, Holden-Day Inc., San Francisco, CA, 525 pp.

Katul G. G., Goltz, S. M., Hsieh, C. I., Cheng, Y., Mowry, F., and Sigmon, J.: 1995, 'Estimation of Surface Heat and Momentum Fluxes Using the Flux-Variance Method above Uniform and Non-Uniform Terrain', *Boundary-Layer Meteorol.* **74**, 237–260.

Kroon, L. J. M. and Bink, N. J.: 1996, 'Conditional Statistics of Vertical Heat Fluxes in Local Advection Conditions', *Boundary-Layer Meteorol.* **80**, 49–78.

Ohtaki, E.: 1985, 'On the Similarity in Atmospheric Fluctuations of Carbon Dioxide, Water Vapor and Temperature over Vegetated Fields', *Boundary-Layer Meteorol.* **32**, 25–37.

Roth, M. and Oke, T. R.: 1995, 'Relative Efficiencies of Turbulent Transfer of Heat, Mass, and Momentum over a Patchy Urban Surface', *J. Atmos. Sci.* **52**, 1863–1874.