WATER RESOURCES RESEARCH, VOL. 37, NO. 9, PAGES 2333–2343, SEPTEMBER 2001

Sensible heat flux estimation by flux variance and half-order time derivative methods

Karen H. Wesson, Gabriel Katul, and Chun-Ta Lai

School of the Environment, Duke University, Durham, North Carolina

Abstract. This study is the first to contrast two similarity theory methods, the flux variance and the half-order time derivative, over a wide range of atmospheric stability and surface roughness conditions. These two methods were selected because they require only single-level temperature measurement to estimate sensible heat flux. The data used were collected over bare soil, a grass-covered forest clearing, and an even-aged pine forest. For all three sites the flux variance method estimated the sensible heat flux relatively well for unstable atmospheric conditions. The half-order time derivative method was found to be sensitive to the parameterization of the eddy diffusivity, especially for the grass and bare soil field sites. Overall, the flux variance method was able to reproduce the measured sensible heat flux with greater accuracy than the half-order time derivative methods for the three experiment sites.

1. Introduction

Exploration of practical methods for estimating sensible heat flux H continues to be an important research topic in surface hydrology [*Kustas et al.*, 1994; *Wang and Bras*, 1998; *Asanuma and Brutsaert*, 1999]. This is primarily because measuring sensible heat flux by eddy correlation methods on a routine basis requires a high-maintenance, stationary setup that is biased by certain wind directions, orientation, and sensor alignment. Similarity theory methods are promising in that they allow the estimation of H with a minimum number of routinely measured meteorological variables. Because indirect methods are based on Monin-Obukhov similarity theory (MOST), they are limited by conditions for which this theory was constructed, primarily the requirements of stationarity and planar homogeneity in flow statistics and the absence of flux divergences.

It is the objective of this study to evaluate, for a variety of atmospheric stability and surface roughness conditions, the performance of two indirect methods, the flux variance method pioneered by *Tillman* [1972] and the recent half-order time derivative method of *Wang and Bras* [1998], in estimating the sensible heat flux. Both of these methods enhance the simplicity of indirect methods by requiring only a single-level temperature measurement to estimate the flux. The flux variance method has been employed in many studies [e.g., *Wesley*, 1988; *Weaver*, 1990; *Lloyd et al.*, 1991; *DeBruin et al.*, 1991, 1993; *Padro*, 1993; *Kustas et al.*, 1994; *Albertson et al.*, 1995; *Kroon and DeBruin*, 1995; *Paw U et al.*, 1995; *Katul et al.*, 1995, 1996; *Hsieh et al.*, 1996; *Andreas et al.*, 1998; *Asanuma and Brutsaert*, 1999] for both homogeneous and inhomogeneous surfaces. However, since the half-order time derivative method was recently proposed, there has not been an in-depth analysis of the model's performance over a variety of homogeneous and inhomogeneous terrain or less ideal meteorological conditions. In particular, the half-order time derivative method and the flux variance method have not been compared, over a wide range of atmospheric stability and roughness conditions, using the same data sets. Therefore it is envisaged that this study will provide a quantitative assessment of the general applicability of these two indirect methods for such a wide range of conditions as well as contrast their relative performance on the same data set.

Paper number 2001WR900021.
0043-1397/01/2001WR900021$09.00

2. Theory

2.1. Flux Variance Method

On the basis of the *Monin and Obukhov* [1954] similarity theory (MOST) any dimensionless turbulence statistic depends only upon the atmospheric stability, $\zeta = z/L$, where z is the height above the displacement height, d_0, and L is the Obukhov length. Using this dimensionless scaling, it can be shown that the variance in air temperature, σ_T, can be expressed as a function of ζ such that

$$\sigma_T/T_* = f(-\zeta), \tag{1}$$

where T_* is the temperature scaling parameter defined by

$$T_* = \langle w'T' \rangle / u_* \tag{2}$$

and L is the Obukhov length given by

$$L = -\frac{u_*^3 T}{\kappa g \langle w'T' \rangle}, \tag{3}$$

where $\langle w'T' \rangle$ is the mean sensible heat flux, $\langle \quad \rangle$ is time averaging, T is the mean atmospheric temperature, κ (=0.4) is von Karman's constant, g is the acceleration due to gravity, and u_* is the friction velocity given by

$$u_* = \sqrt{-\langle u'w' \rangle}, \tag{4}$$

where u' and w' are the turbulent fluctuations in longitudinal and vertical velocities, respectively. As discussed by *Albertson et al.* [1995], the function $f(-\zeta)$ in (1) must satisfy two limits: (1) In the neutral case, $-\zeta \rightarrow 0$ and σ_T/T_* approaches a constant. (2) In the free convection limit, $-\zeta \rightarrow \infty$ and σ_T/T_*

Table 1. Required Measured Input Variables and Model Parameters for the Flux Variance and the Half-Order Time Derivative Methods

Method	Measured Variables	Model Parameters
Flux variance	T, σ_T, u_* or u, and R_n	C_1, C_3
Half-order time derivative	T and u_* or u	$K_H(\alpha, \gamma_1, \gamma_2)$

should be independent of u_*. These two limits can be satisfied with the expression

$$\sigma_T/T_* = C_1(C_2 - \zeta)^{-1/3}, \tag{5}$$

where C_1 and C_2 are similarity constants. For the free convection (and strongly unstable) case, (5) can be approximated as

$$\sigma_T/T_* = C_1(-\zeta)^{-1/3}, \tag{6}$$

where $C_1 = 0.99$ (corrected for $\kappa = 0.4$) as determined by *Wyngaard et al*. [1971]. From this it can be shown that the sensible heat is related to σ_T by

$$H = \rho c_p \langle w'T' \rangle = (C_1)^{-3/2}\sigma_T^{3/2}T^{-1/2}\rho c_p(\kappa g z)^{1/2}, \tag{7}$$

where ρ is the mean air density and c_p is the specific heat of air. Equation (7) assumes that near-convective conditions exist, and it is therefore limited by its failure to take into account changing atmospheric stability conditions toward neutral conditions. In addition, (7) cannot predict negative sensible heat flux since $\sigma_T \geq 0$. To allow the flux variance method to take into account negative sensible heat flux, the approximations

$$\sigma_T/T_* = C_3 \qquad H = -\rho c_p \frac{\sigma_T u_*}{C_3} \tag{8}$$

are used for stable runs, where C_3 is another similarity constant. Here the net radiation, R_n, can be used as a surrogate for identifying nighttime atmospheric conditions, and typical reported values for C_3 are 2.4, 3.5, and 4.0 [*Stull*, 1988].

2.2. Half-Order Time Derivative Method

Assuming downgradient transport of heat and first-order closure for the turbulent sensible heat flux,

$$H(z, t) = -\rho c_p K_H \frac{\partial T}{\partial z}, \tag{9}$$

and within the surface layer, the vertical transfer of heat can be described by a one-dimensional diffusion equation [e.g., *Priestly*, 1959],

$$\frac{\partial T(z, t)}{\partial t} = \frac{\partial}{\partial z}\left(K_H \frac{\partial T}{\partial z}\right), \tag{10}$$

when sufficient horizontal uniformity exists under the boundary conditions

$$T = T_\infty, \quad t > 0, \quad z \to \infty \qquad T = T_\infty, \quad t = 0, \quad z > 0, \tag{11}$$

where K_H is the eddy diffusivity for heat, t is integration time, and the other symbols bear their usual meanings. Equation (9) represents the flux gradient method and is based on the Monin-Obukhov similarity theory which says that the average vertical flux of a quantity is proportional to the vertical gradient of the time-averaged quantity. To compute H from (9), at least two measurements of T are needed which adds complexity to the use of the flux gradient method. However, recently, a new method proposed by *Wang and Bras* [1998], called the half-order time derivative method, employs fractional calculus to rearrange (10) such that the vertical gradient of temperature can be expressed as a weighted average of the air temperature time series at a given height (see Appendix A for derivation). Applying this to (9),

$$H(z, t) = -2\rho C_\rho \sqrt{\frac{K_H}{\pi}} \int_0^t \frac{\partial T(z, s)}{\partial s}\, d(\sqrt{t - s}). \tag{12}$$

where s is the integration variable.

For this study, we adopt the most commonly used parameterization of K_H,

$$K_H = \kappa u_* z \phi_h^{-1}, \tag{13}$$

where ϕ_h is the stability correction function given by *Businger et al*. [1971] as

$$\phi_h = \begin{cases} \alpha(1 - \gamma_1\zeta)^{-1/2}, & \zeta < 0 \\ \alpha, & \zeta = 0 \\ \alpha + \gamma_2\zeta, & \zeta > 0, \end{cases} \tag{14}$$

where α, γ_1, and γ_2 are empirical constants. From several field experiments, α has been measured to be ~0.74. The other constants, γ_1 and γ_2, are equal to ~9 and ~5, respectively [*Businger et al*., 1971]. For measurements taken close to the surface (i.e., $z \approx 1$ m), *Wang and Bras* [1998] adopted an alternative parameterization, $K_H \approx uz$ (erroneously listed as u by *Wang and Bras* [1998]), where u is the mean longitudinal wind velocity.

The substitution of (10) with the formulation of K_H given by (13) into (9) contradicts the conditions under which (9) holds, since (13) was derived for nonstationary conditions and (9) assumes stationarity. In addition, (12) was arrived at upon assuming K_H is independent of z. Therefore, given the approximations made in deriving the half-order time derivative method, (12) can only be treated as an approximate solution of (9).

2.3. Model Application

Because of the conditions under which the half-order time derivative method was developed, the integration should be initiated when $H \approx 0$. In addition, (12) requires a relatively continuous data set of T and u to estimate H. These two requirements can prevent a continuous estimation of H since a gap in measured T of more than a couple hours will require that the calculation of H be ended and reinitiated only when $H \approx 0$. Another complexity of implementing the half-order time derivative method occurs when (13) is used in the formu-

Table 2. Summary of Site Characteristics, Instrument Heights, and Measured Meteorological Variables for the Four Experiments

Description	Experiment 1	Experiment 2	Experiment 3	Experiment 4
Surface type	pine forest	grass	grass	bare soil, desert
Location	Duke Forest Blackwood Division, Durham, North Carolina	Duke Forest Blackwood Division, Durham, North Carolina	Duke Forest Blackwood Division, Durham, North Carolina	Owens Valley, California
Date of experiment	August 1998, October–December 1998, and January–April 1999	May 1997	August 1999	June–July 1993
Canopy height, m	14.0	1.0	1.0	···
Estimated momentum roughness z_0, cm	120	10	10	0.013
Estimated zero-plane displacement height d_0, m	9.3	0.7	0.7	···
Measurement height, m	14.5	2.8	2.87	1.0
Instruments	CSAT3 3-D sonic anemometer and KH20 Hygrometer	Gill triaxial sonic anemometer and KH20 Hygrometer	CSAT3 3-D sonic anemometer and KH20 Hygrometer	Gill triaxial sonic anemometer and KH20 Hygrometer
Sampling frequency, Hz	5	10	10	56
Sampling period, min	30	20	20	15

lation of K_H since it makes (12) nonlinear in H. Therefore, in this case, iteration is required to estimate H.

The input variables and model parameters for each of the two methods are listed in Table 1. For the half-order time derivative method, only measurement of T and u_* (or u, depending on the formulation of K_H) is required. However, the flux variance method requires the measurement of T, σ_T, u_*, and R_n to estimate H. In terms of measured variables the increased complexity of the flux variance method over the half-order time derivative method is due to the fact that the flux variance method is unable to calculate a negative H, and therefore the approximation of (8) must be implemented. This requires R_n to identify stable atmospheric conditions and u_* to estimate H. (However, if necessary, u_* can be estimated from u based on MOST.)

3. Experiment

Micrometeorological and sensible heat flux measurements used for this study were collected at three sites: (1) a pine forest, (2) a grass-covered forest clearing, and (3) the bare soil of dry Owens Lake bed. Site characteristics are summarized in Table 2 for all three sites. (Note that measurements were taken at site 2 in May 1997 and again in August 1999.) The pine forest is located in the Blackwood Division of the Duke Forest in Durham, North Carolina. The stand is composed of even-aged *Pinus taeda L.* (loblolly pine) and stretches 300–600 m in the east-west direction and 1000 m in the north-south direction. The grass-covered clearing is also located within the Blackwood Division of the Duke Forest, near the pine forest of site 1, and is approximately 480 m by 305 m. The third site is the dry Owens Lake bed located in Owens Valley, California, between the Sierra Nevada and White and Inyo Mountains. The surface of the basin is composed of crusted sand with a substantial amount of evaporated salts.

The three sites were chosen for this study to provide a variety of surface roughness, atmospheric stability conditions, and land cover types. Table 2 gives a summary of the site characteristics, instrument heights, and measured meteorological variables. Table 3 shows the mean, minimum, and maximum ranges of the key meteorological variables used in the two models. Further details about the sites and experimental setup are given by *Katul* [1994] and *Albertson et al.* [1995] for dry Owens Lake bed, *Katul et al.* [1995] and *Lai and Katul*

Table 3. Range of Values and Mean Values of the Five Major, Measured Meteorological Variables Used in the Flux Variance and Half-Order Time Derivative Methods

Surface Cover	T, °C	H, W m^{-2}	u, m s^{-1}	u_*, m s^{-1}	σ_T, °C
Pine forest					
Maximum	35.76	371.64	4.88	1.06	1.48
Minimum	0.47	−96.43	0.024	0.014	0.032
Mean	3.69	5.79	1.05	0.90	0.91
Grass-covered clearing (1997)					
Maximum	36.47	446.42	3.77	0.63	2.14
Minimum	7.06	−83.95	0.012	~0.0	~0.0
Mean	21.60	48.91	1.12	0.16	0.43
Grass-covered clearing (1999)					
Maximum	38.14	274.97	2.44	0.40	1.46
Minimum	23.60	−37.39	0.021	~0.0	~0.0
Mean	30.36	38.59	0.29	0.15	0.40
Dry Owens Lake bed					
Maximum	36.93	443.0	11.25	0.58	2.04
Minimum	13.67	−74.35	0.52	0.04	0.10
Mean	25.93	70.95	4.31	0.25	0.78

Table 4. Number of Data Points Used in the Models for Each of the Four Experiments

Site Description	Number of Points
Pine forest	3796
Grass-covered clearing (1997)	1850
Grass-covered clearing (1999)	225
Dry Owens Lake bed	325

[2000] for the grass-covered forest clearing, and *Katul et al.* [1999], *Katul and Albertson* [1999], and *Lai et al.* [2000a, 2000b] for the pine forest.

4. Analysis and Results

To compute H using the half-order time derivative method, a continuous data set of T measurements is needed. Therefore, since some gaps exist in all four data sets, a linear interpolation was performed to compute the missing measured meteorological variables for up to three time intervals in 30 min incremented data and up to four intervals in 20 min incremented data. When the size of the gaps in the data exceeded three to four missing intervals, the computation of the sensible heat flux was terminated and restarted again when the measured sensible heat flux was ~0, as required by the half-order time derivative method. To be consistent, the data used for the flux variance and flux gradient methods were made identical to those identified for the half-order time derivative method. The number of points used for each site is shown in Table 4.

4.1. Flux Variance Method

Using measured meteorological variables, H is estimated using the flux variance method and is presented in Figure 1 for the four experiments. For "daytime conditions," when the measured H was greater than zero, H was estimated using (7) with $C_1 = 0.99$ (corrected for κ =0.4) as determined by *Wyngaard et al.* [1971] for the free convective case. During "nighttime conditions," when values of measured H were <0, (8) was used instead. All three measured values of C_3 (2.4, 3.5,

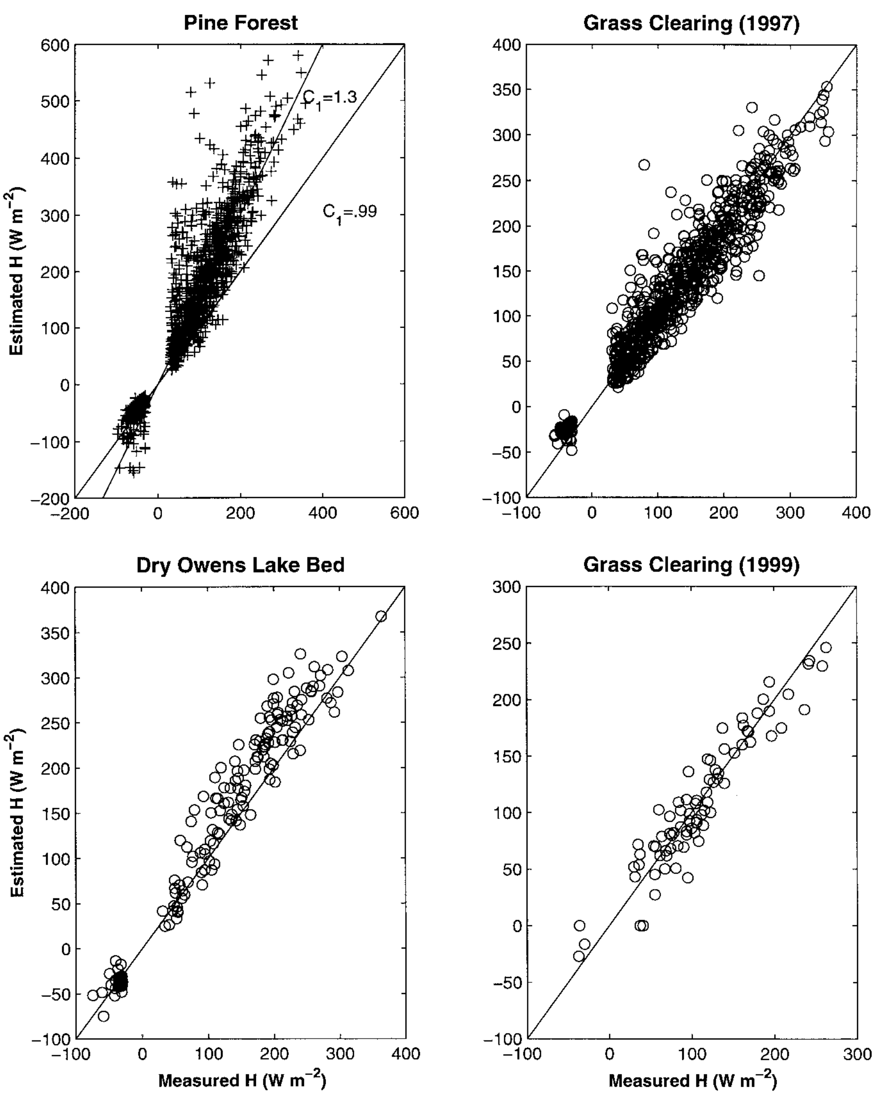

Figure 1. Sensible heat flux (H) estimated using the flux variance method for the four experiments. For the pine forest, two solid lines are displayed: one being the 1:1 fit between measured and modeled H with $C_1 = 0.99$ and the second illustrating the optimized regression line between measured and estimated H yielding $C_1 = 1.3$.

Table 5. Regression Slope and y Intercept, Correlation Coefficient (r), and Root-Mean-Square Error for the Estimated Sensible Heat Flux Using the Flux Variance Method From Equation (8) to Estimate Negative Sensible Heat Flux for the Three Measured Values of C_3 (2.4, 3.5, and 4.0) for the Four Experiments

Experiment	C_3	Regression Slope	Intercept	r	RMSE,[a] W m^{-2}
Pine forest	2.4	1.4602	−7.2688	0.4241	49.5629
Pine forest	3.5	1.0013	−4.9843	0.4241	28.1482
Pine forest	4.0	0.8761	−4.3613	0.4241	24.3165
Grass-covered clearing (1997)	2.4	0.4559	−24.3398	0.2722	11.5650
Grass-covered clearing (1997)	3.5	0.3126	−16.6901	0.2722	12.8816
Grass-covered clearing (1997)	4.0	0.2736	−14.6039	0.2722	15.4546
Grass-covered clearing (1999)	2.4	0.2391	−15.9435	0.0406	21.4482
Grass-covered clearing (1999)	3.5	0.1640	−10.9327	0.0406	22.2047
Grass-covered clearing (1999)	4.0	0.1435	−9.5661	0.0406	23.1098
Dry Owens Lake bed	2.4	0.8772	−29.2980	0.4896	29.3170
Dry Owens Lake bed	3.5	0.6015	−20.0901	0.4896	12.6096
Dry Owens Lake bed	4.0	0.5263	−17.5788	0.4896	10.6452

[a]RMSE, root mean square error.

and 4.0) were used in (8), and the method performance is statistically contrasted in Table 5 for the four experiments. The accuracy of (8) in estimating negative H is not significantly different for any of the three measured values of C_3 for any of the four experiments. We choose to let $C_3 = 4.0$ for further statistical analysis of the flux variance method based on the slightly better performance at dry Owens Lake bed of this value over the other two measured values (2.4 and 3.5). The dry Owens Lake bed was chosen as the reference site since it is the most planar homogeneous site of the three presented in this study.

The statistics for the flux variance method, for both daytime and nighttime conditions combined, are shown in Table 6. Because (7) is not defined as $T_* \rightarrow 0$, when the values of the measured H were near zero (for practical purposes we set it at ± 25 W m^{-2}), the estimated values of H were not used in the statistical analysis. (These values of H are also not shown in Figure 1.) For three of the experiments, the experiment at dry Owens Lake bed and the 1997 and 1999 experiments at the grass-covered clearing, the flux variance method estimates H well with correlation coefficients of 0.97, 0.95, and 0.95, respectively. However, for the pine forest the flux variance method overestimates H. To "calibrate" this method, the value of C_1 is increased until the performance of the model is optimized. It is found that at a value of $C_1 = 1.3$ the flux variance method accurately reproduces the measured H with a correlation coefficient of 0.93. The accuracy of this fit is shown graphically in Figure 1 and statistically in Table 6. The value of $C_1(1.3)$ found in this experiment is greater than the values of C_1 cited in similar studies over heterogeneous surfaces. This is attributed to the close proximity of the instruments to the pine vegetation, which would have caused the instruments to sense the added spatial inhomogeneities in the heat sources and to attribute this added variance to an increase in sensible heat via (7). This will be discussed further in section 5.

4.2. Half-Order Time Derivative Method

The H estimated using the half-order time derivative method and the MOST parameterization of the eddy diffusivity from (13) is shown in Figure 2. For the pine forest experiment the half-order time derivative method seems to reproduce the 1:1 response of measured sensible heat fluxes, though there is considerable scatter in the comparison. (See Table 6.) However, for the grass-covered clearing and dry Owens Lake bed, H is significantly underestimated.

Figure 3 shows H estimated at the grass site using the half-order time derivative method with $K_H \approx uz$ (as proposed by *Wang and Bras* [1998] for low height z). While the proposed correction (predominantly an increase) in the eddy diffusivity adjusts the predicted values of H in the correct direction, the method still underestimates the measured fluxes for large values of H. The observation is the same when this method is applied to the measurements from the dry Owens Lake bed. In Figure 4, H is estimated for dry Owens Lake bed using the half-order time derivative method with $K_H \approx uz$ and using the flux gradient method. Again, while, on average, the half-order time derivative method with the suggested correction to the parameterization of K_H seems to improve predicted H, the method fails at large values of H.

In contrast, the flux gradient method of (9) (with measured surface and air temperature and the parameterization of heat roughness height as described by *Cahill et al.* [1997]) shows good agreement between measured and estimated sensible heat flux. We iterate that the parameterization of the eddy diffusivity used for the flux gradient method is that given by MOST and not the approximation $K_H \approx uz$. This latter anal-

Table 6a. Regression Slope and y Intercept, Correlation Coefficient (r), and Root-Mean-Square Error for the Estimated Sensible Heat Flux Using the Flux Variance Method

Experiment	Method	Slope	Intercept	R	RMSE, W m^{-2}
Pine forest	flux variance ($C_1 = 0.99$)	1.4270	26.5693	0.9334	87.6125
Pine forest	flux variance ($C_1 = 1.3$)	1.0099	8.2854	0.9337	39.6790
Grass-covered clearing (1997)	flux variance	0.8997	15.1112	15.1112	27.3244
Grass-covered clearing (1999)	flux variance	0.9088	9.1133	9.1133	20.4863
Dry Owens Lake bed	flux variance	1.0948	9.7455	9.7455	35.0840

Table 6b. Regression Slope and y Intercept, Correlation Coefficient (r), and Root-Mean-Square Error for the Estimated Sensible Heat Flux Using the Half-Order Time Derivative Method and the Flux Gradient Method for Different Parameterizations of the Eddy Diffusivity for the Four Experiments

Experiment	Method	Slope	Intercept	r	RMSE, W m^{-2}
Pine forest	half-order time derivative method ($K_H = u_* kz$)	0.5215	9.3257	0.6855	62.3881
Pine forest	half-order time derivative method ($K_H = uh/3$)	0.8670	−12.8907	0.5651	96.4428
Grass-covered clearing (1997)	half-order time derivative method ($K_H = u_* kz$)	0.1842	2.6430	0.7348	80.9934
Grass-covered clearing (1997)	half-order time derivative method ($K_H = uz$)	0.7290	−14.7299	0.7515	65.7158
Grass-covered clearing (1997)	half-order time derivative method ($K_H = Uz$)	0.6923	−24.9742	0.7020	76.9421
Grass-covered clearing (1999)	half-order time derivative method ($K_H = u_* kz$)	0.6464	6.9728	0.7884	41.3232
Grass-covered clearing (1999)	half-order time derivative method ($K_H = uz$)	0.7721	−12.9882	0.7786	48.8907
Grass-covered clearing (1999)	half-order time derivative method ($K_H = Uz$)	0.7409	−20.6874	0.7847	52.1124
Dry Owens Lake bed	half-order time derivative method ($K_H = u_* kz$)	0.1951	−4.4867	0.8794	96.3913
Dry Owens Lake bed	half-order time derivative method ($K_H = uz$)	0.9690	−42.7342	0.8317	77.9119
Dry Owens Lake bed	half-order time derivative method ($K_H = Uz$)	1.1174	−41.6897	0.8141	86.7060
Dry Owens Lake bed	flux gradient	1.6958	−36.5949	0.9420	91.2518

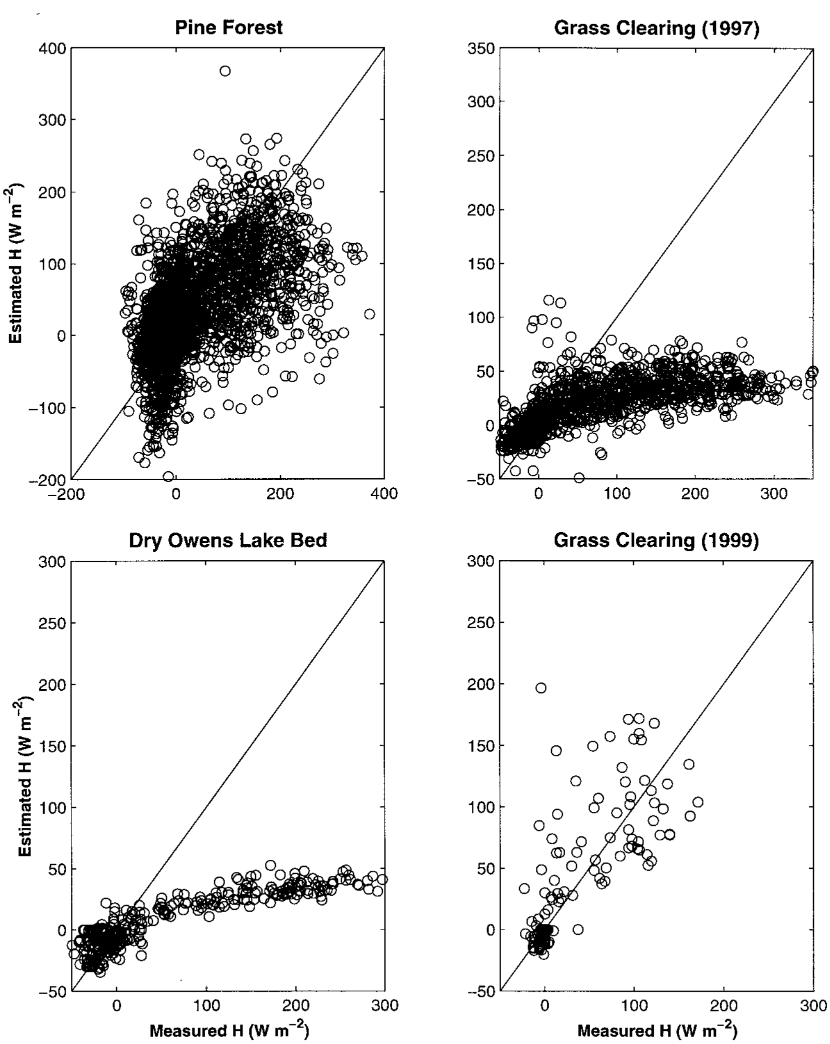

Figure 2. Sensible heat flux estimated for the four experiments using the half-order time derivative method and the Monin-Obukhov similarity theory (MOST) parameterization of the eddy diffusivity. The solid line is 1:1.

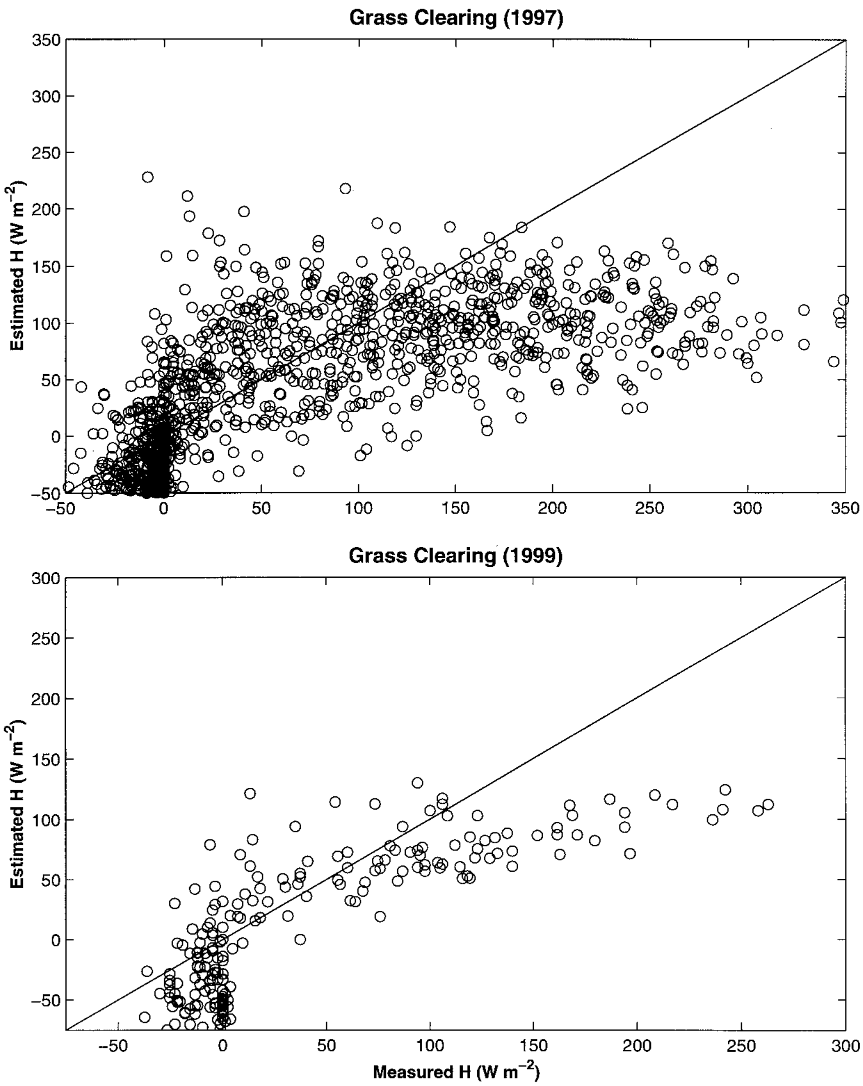

Figure 3. Estimated sensible heat flux over the grass-covered clearing using the half-order time derivative method with the approximation that $K_H \approx uz$. The solid line is 1:1.

ysis demonstrates that the poor performance of the half-order time derivative method is not due to the MOST parameterization of K_H but rather is due to the assumptions of near-constant diffusivity in its derivation.

5. Discussion and Conclusions

Assuming the atmospheric surface layer (ASL) extends to 2–3 times h [*Parlange and Brutsaert*, 1993], the measurements at both the grass and desert sites were carried out in the ASL while the data from the pine forest were taken in the canopy sublayer (CSL). Given this, the results indicate that the flux variance method reproduces the measured sensible heat flux well in the ASL, though in the CSL the method overestimates the measured sensible heat flux. This overestimation is due to the fact that the temperature variance is not dependent only on the sensible heat flux. Instead, the inhomogeneity of the heat sources can increase the temporal variance. This added variance has been found in many studies to increase the value of C_1 in the CSL [e.g., *Padro*, 1993; *Katul et al.*, 1995, 1999; *Andreas et al.*, 1998]. In this study, a value of $C_1 = 1.3$ allows the flux variance method to fit the pine forest data with a correlation coefficient of 0.93. Compared to values of C_1 in the literature, 0.92 [*Monji*, 1973; *Högström and Smedman-Högström*, 1974], 0.95 [*Ohtaki*, 1985; *Katul et al.*, 1995; *Katul and Hsieh*, 1997], 0.97 [*Högström*, 1990; *Albertson et al.*, 1995], 0.99 [*Wyngaard et al.*, 1971], 1.0 and 1.1 [*Kader and Yaglom*, 1990], 1.1 [*Kustas et al.*, 1994], and 1.25 [*Wesley*, 1988], the value of $C_1(1.3)$ found in this study is large. However, as mentioned before, the measurements in the pine forest were performed near the canopy-atmosphere interface. This closer proximity to the vegetation lead to increased sensitivity to the inhomogeneities of the heat sources, which, in turn, lead to a higher value of C_1 than cited in the literature.

For the half-order time derivative method the results show that the method is hypersensitive to the parameterization of the eddy diffusivity. When using MOST to estimate the diffusivity, the method significantly underestimates H for both the grass-covered clearing and the dry Owens Lake bed. Since the measurements for both of these sites were collected in the ASL, the MOST diffusivity parameterization is more appro-

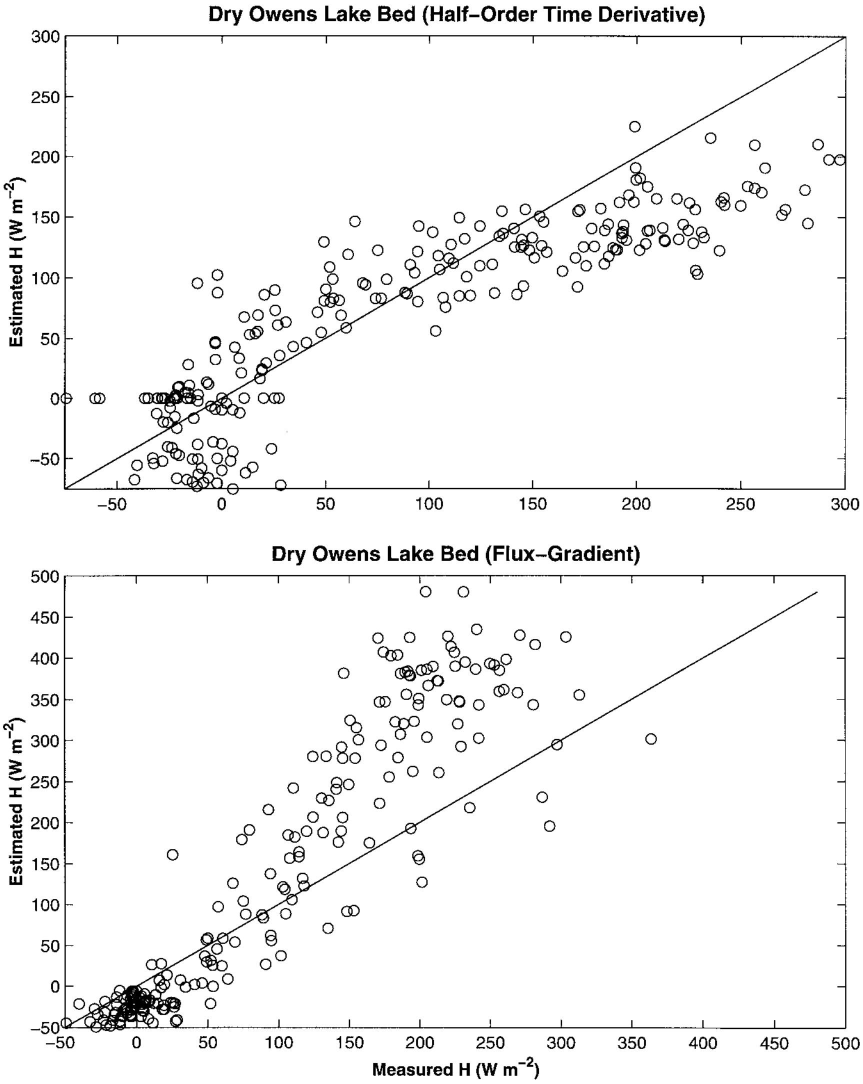

Figure 4. Estimated sensible heat flux over dry Owens Lake bed using the half-order time derivative method with the approximation that $K_H \approx uz$, and using the flux gradient method with MOST parameterization for K_H. The solid line is 1:1.

priate when compared to the proposed $K_H \approx uz$. In fact, for the three sites, 81–100% of the measured values of z/L fall within the range stipulated by *Businger et al.* [1971] for the application of the MOST parameterization of (13) (i.e., 0–2 for stable conditions and −2–0 for unstable cases). (See Table 7.) To test whether the underestimation of H is site-specific, the flux gradient method with MOST parameterization of K_H was applied to the data from dry Owens Lake bed as described by *Cahill et al.* [1997]. The results indicate that there is good agreement between the observed and measured H for the flux gradient method. Therefore the underestimation of H is due to the chosen form of the parameterization of K_H in the half-order time derivative method for the two sites.

Table 7. Percentage of Measured Values of z/L for Each Data Set for Which the Values of z/L Fall Between the Range of 2 and −2 as Stipulated by *Businger et al.* [1971]

Experiment	Measured z/L Between 2 and −2, %
Pine forest	88
Grass-covered clearing (1997)	81
Grass-covered clearing (1999)	84
Dry Owens Lake bed	100

To further analyze this, the proposed correction to the eddy diffusivity (i.e., $K_H \approx uz$) of *Wang and Bras* [1998] is implemented for the half-order time derivative method for the data from the grass-covered clearing and the dry Owens Lake bed. The results show that the comparison between the measured and estimated H is improved. However, the method fails to correctly estimate H for large measured values of H. *Wang and Bras* [1998] also suggest that instead of the using the time series of u a characteristic wind speed, U, reflecting a longer-term u should be used such that $K_H \approx Uz$. When this suggestion is applied to the method for the data from the three sites, letting U equal the average value of u, the accuracy of the method is not significantly changed. (See Table 6.) Because of this and the uncertainty of what a "characteristic" mean wind speed should be in a semiempirical method, the correction to the eddy diffusivity may simply be kept as $K_H \approx uz$. Even with this

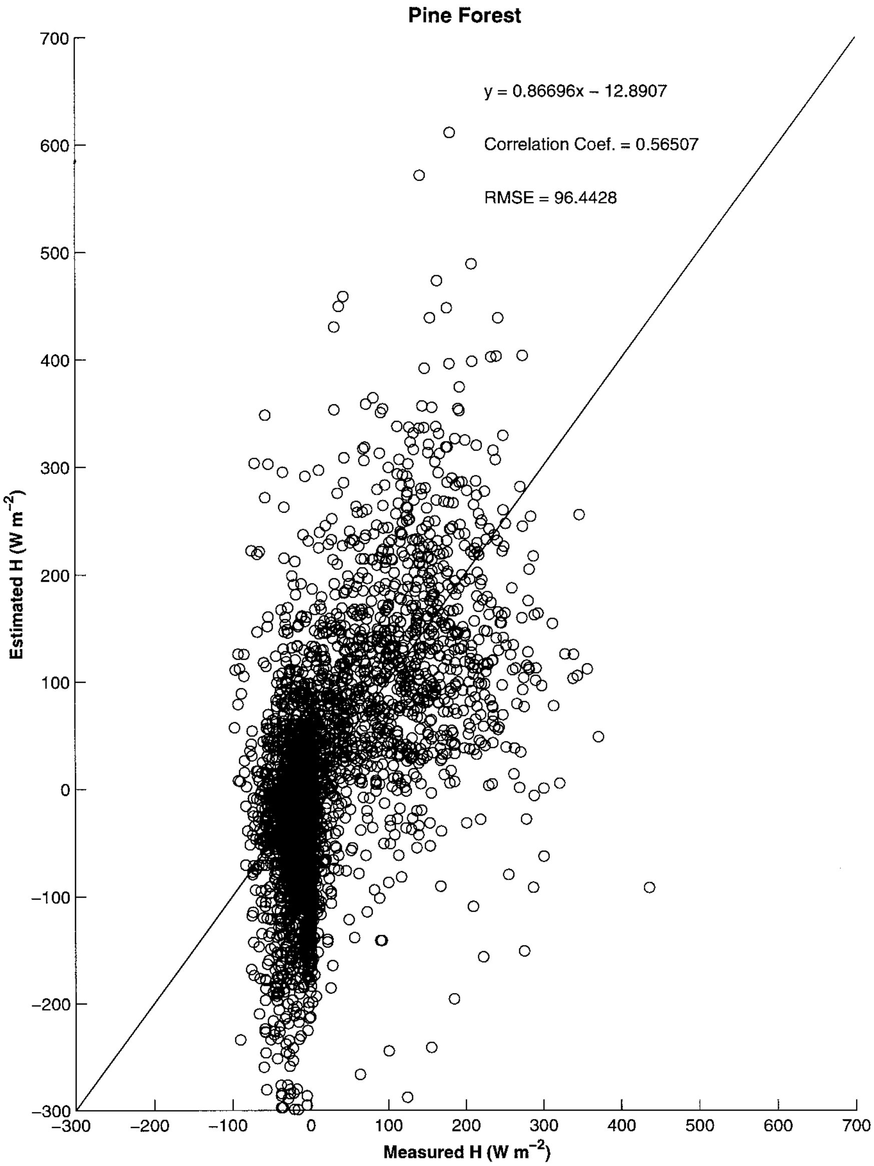

Figure 5. Estimated sensible heat flux over the pine forest using the half-order time derivative method with the approximation that $K_H \approx hu/3$. The solid line is 1:1.

diffusivity, the agreement between measured and estimated H is still inferior to that estimated by the flux variance method for the two sites.

For the pine forest the half-order time derivative method with the MOST parameterization of K_H, as given in (13), reproduces the measured H well (at least when contrasted to the other two sites). This result, at first glance, is rather surprising since the measurements at this site were carried out in the CSL where MOST is less likely to hold. However, between 1 and 2 times the canopy height of the forest, and for neutral conditions, the characteristic length scale of the diffusivity is invariant to changes in z [*Kaimal and Finnigan*, 1994]. Therefore the invariance of K_H to height variation within this region is perhaps more consistent with the approximately constant diffusivity in the derivation of (12). This implies that the MOST parameterization of K_H could be replaced by the suggested approximation $K_H \approx (h/3)u$. The characteristic length scale of $h/3$ is derived from the mixing layer analogy of *Raupach et al.* [1996] for a generic canopy and is consistent with measured cospectral peaks of w' and T' near $z/h = 1$ for the pine stand [*Katul et al.*, 1998]. The result of implementing this approximation is shown in Figure 5. Statistical analysis indicates that the half-order time derivative method with the approximation $K_H \approx (h/3)u$ is comparable in accuracy to the same method with the MOST parameterization of K_H (see Table 6). (While the approximation $K_H \approx (h/3)u$ predicts the measured sensible heat flux better in terms of the regression slope, there is more scatter in the data than in the predictions of the same method with the MOST parameterization of K_H.)

The results from applying the half-order time derivative method to predict H at the three sites, the grass-covered clearing, dry Owens Lake bed, and the pine forest, indicate that the method is very sensitive to the parameterization of the eddy diffusivity. For all four experiments the suggested parameterization $K_H \approx uz$ or $(h/3)u$ performs as well as or better than the MOST parameterization of K_H. This result is somewhat surprising since the parameterization $K_H \approx uz$ or $(h/3)u$ is an empirical one. It seems that the better performance of this parameterization is more an artifact of the method than of the diffusivity model itself. This is evident from the good performance of the flux gradient method with the MOST parameterization of K_H when applied to the data from dry Owens Lake bed.

For this study, the performance of the half-order time derivative method was found to be best when the sensible heat flux was relatively low or when the conditions at the measurement site were such that the characteristic length scale of the eddy diffusivity was invariant to changes in z. The flux variance method, however, predicted the sensible heat flux well for all of the sites in this study. Therefore, in comparing the flux variance method and the half-order time derivative methods for a variety of surface and atmospheric stability conditions, we conclude that the flux variance method predicts the sensible heat flux better than the half-order time derivative method.

Appendix A: Algorithm for the Half-Order Time Derivative

Much of the derivation below is presented by *Wang and Bras* [1998], but the key assumptions are repeated for completeness. For a horizontally homogeneous, turbulent flow the nonstationary vertical transfer of heat can be described by a one-dimensional diffusion equation [e.g., *Priestly*, 1959]:

$$\frac{\partial T(z, t)}{\partial t} = \frac{\partial}{\partial z}\left(K_H \frac{\partial T}{\partial z}\right). \tag{A1}$$

Equation (A1) is constrained by the upper boundary condition

$$T = T_\infty, \qquad t > 0, \qquad z \to \infty \tag{A2}$$

and initial condition

$$T = T_\infty, \qquad t = 0, \qquad z > 0. \tag{A3}$$

Assuming that K_H is constant and performing a change of variables, $\Theta = T - T_\infty$ and $u = z/\sqrt{K_H}$, (A1) reduces to

$$\frac{\partial \Theta(u, t)}{\partial t} = \frac{\partial^2 \Theta(u, t)}{\partial u^2}, \tag{A4}$$

with the upper boundary condition

$$\Theta = 0, \qquad t = 0, \qquad u < 0 \tag{A5}$$

and initial condition

$$\Theta = 0, \qquad t > 0, \qquad u \to \infty. \tag{A6}$$

The succeeding derivation follows *Oldham and Spanier* [1974] and *Spanier* [1977]. Laplace transformation of (A4) with the application of (A5) leads to the relationship

$$\frac{\partial^2 \bar{\Theta}(u, s)}{\partial u^2} = s\bar{\Theta}(u, s), \tag{A7}$$

where $\bar{\Theta}$ represents the Laplace transform of Θ defined as

$$\bar{\Theta}(u, s) = \int_0^\infty \exp(-st)\Theta(u, t)\, dt. \tag{A8}$$

The general solution of (A7) is

$$\bar{\Theta}(u, s) = A(s) \exp(u\sqrt{s}) + B(s) \exp(-u\sqrt{s}), \tag{A9}$$

where A and B are arbitrary functions of s. From the boundary condition in (A6) it follows that $A(s) = 0$ and

$$\bar{\Theta}(u, s) = B(s) \exp(-u\sqrt{s}). \tag{A10}$$

Differentiating with respect to u gives

$$\frac{\partial \bar{\Theta}(u, s)}{\partial u} = -\sqrt{s}B(s) \exp(-u\sqrt{s}), \tag{A11}$$

and substituting (A10) into (A11) yields

$$\frac{\partial \bar{\Theta}(u, s)}{\partial u} = -\sqrt{s}\bar{\Theta}(u, s). \tag{A12}$$

Utilizing the identity

$$\overline{\frac{d^{1/2}f}{dt^{1/2}}} = \sqrt{s}\bar{f}, \tag{A13}$$

which is valid for most functions f, the relationship

$$\frac{\partial \bar{\Theta}(u, t)}{\partial u} = -\frac{\partial^{1/2}\Theta(u, t)}{\partial t^{1/2}} \tag{A14}$$

is obtained. Changing back to the original variables, (A14) becomes

$$\frac{\partial T(z, t)}{\partial z} = -\frac{1}{\sqrt{K_H}} \frac{\partial^{1/2}}{\partial t^{1/2}}[T(z, t) - T_\infty]. \tag{A15}$$

The definition of a half-order time derivative can be found in many books on fractional calculus [e.g., *Oldham and Spanier*, 1974; *Miller and Ross*, 1993]. When $f = 0$, the half-order time derivative can be defined accordingly

$$\frac{d^{1/2}f(t)}{dt^{1/2}} = \frac{1}{\sqrt{\pi}} \int_0^t f'(s) \frac{ds}{\sqrt{t-s}}. \tag{A16}$$

Inserting this definition for the half-order time derivative in (A15) leads to

$$\frac{\partial T(z, t)}{\partial z} = -\frac{1}{\sqrt{\pi K_H}} \int_0^t \frac{\partial T(z, s)}{\partial s} \frac{ds}{\sqrt{t-s}}, \tag{A17}$$

which, with a change in variables, allows the vertical gradient of temperature to be expressed as

$$\frac{\partial T(z, t)}{\partial z} = -\frac{2}{\sqrt{\pi K_H}} \int_0^t \frac{\partial T(z, s)}{\partial s} d(\sqrt{t-s}). \tag{A18}$$

H can be estimated by substituting (A18) into (9) such that

$$H(z, t) = -\rho c_p K_H \frac{\partial T}{\partial z} = -2\rho c_p \sqrt{\frac{K_H}{\pi}} \int_0^t \frac{\partial T(z, s)}{\partial s} d(\sqrt{t-s}). \tag{A19}$$

Equation (A19) can be numerically integrated as follows:

$$H_N = -2\rho c_p \sqrt{\frac{K_H}{\pi}} \sum_{i=0}^{N} \frac{T_{i+1} - T_i}{t_{i+1} - t_i} [\sqrt{t_N - t_{i+1}} - \sqrt{t_N - t_i}], \tag{A20}$$

where N is the number of intervals into which the integration time t is divided. Because of the boundary condition in (A3) the model should start when $H \approx 0$. In addition, using the definition of the half-order time derivative as shown in (A16) assumes that $T_\infty = 0$ at $t = 0$. The overall solution is not very sensitive to this latter approximation.

Acknowledgments. The authors would like to thank Jingfeng Wang for his helpful comments and suggestions. This project was funded, in part, by the National Science Foundation (EAR-99-03471, BIR-95-12333) and the Department of Energy through the FACE-FACTS project and the National Institute for Global Environmental Change (NIGEC) through the Southeast Regional Center at the University of Alabama, Tuscaloosa (DE-FC030-90ER61010). The data sets and Matlab algorithms are available upon request from the first author.

References

Albertson, J. D., M. B. Parlange, G. G. Katul, C.-R. Chu, H. Stricker, and S. Tyler, Sensible heat flux from arid regions: A simple flux-variance method, *Water Resour. Res.*, *31*, 969–973, 1995.

Andreas, E. L., R. J. Hill, J. R. Gosz, D. I. Moore, W. D. Otto, and A. D. Sarma, Statistics of surface-layer turbulence over terrain with metre-scale heterogeneity, *Boundary Layer Meteorol.*, *86*, 379–408, 1998.

Asanuma, J., and W. Brutsaert, Turbulence variance characteristics of temperature and humidity in the unstable atmospheric surface layer above a variable pine forest, *Water Resour. Res.*, *35*, 2281–2288, 1999.

Businger, J. A., J. C. Wyngaard, Y. Izumi, and E. F. Bradley, Flux profile relationships in the atmospheric surface layer, *J. Atmos. Sci.*, *28*, 181–189, 1971.

Cahill, A. T., M. B. Parlange, and J. D. Albertson, On the Brutsaert temperature roughness length model for sensible heat flux estimation, *Water Resour. Res.*, *33*, 2315–2324, 1997.

DeBruin, H., N. Bink, and L. Kroon, Fluxes in the surface layer under advective conditions, in *Land Surface Evaporation—Measurements and Parameterization*, edited by T. J. Schmugge and C. André, pp. 157–169, Springer-Verlag, New York, 1991.

DeBruin, H., W. Kohsiek, and B. Van Den Hurk, A verification of some methods to determine the fluxes of momentum, sensible heat, and water vapour using standard deviation and structure parameter of scalar meteorological quantities, *Boundary Layer Meteorol.*, *63*, 231–257, 1993.

Högström, U., Analysis of turbulence structure in the surface layer with a modified similarity formulation for near neutral conditions, *J. Atmos. Sci.*, *47*, 1949–1972, 1990.

Högström, U., and A. S. Smedman-Högström, Turbulence mechanisms at an agricultural site, *Boundary Layer Meteorol.*, *7*, 373–389, 1974.

Hsieh, C.-I., G. G. Katul, J. Schieldge, J. Sigmon, and K. R. Knoerr, Estimation of momentum and heat fluxes using dissipation and flux-variance methods in the unstable surface layer, *Water Resour. Res.*, *32*, 2453–2462, 1996.

Kader, B. A., and A. M. Yaglom, Mean fields and fluctuation moments in unstable stratified turbulent boundary layers, *J. Fluid Mech.*, *212*, 637–662, 1990.

Kaimal, J., and J. Finnigan, *Atmospheric Boundary Layer Flows: Their Structure and Measurement*, 289 pp., Oxford Univ. Press, New York, 1994.

Katul, G. G., A model for sensible heat-flux probability density-function for near-neutral and slightly-stable atmospheric flows, *Boundary Layer Meteorol.*, *71*, 1–20, 1994.

Katul, G. G., and J. D. Albertson, Modeling CO_2 sources, sinks, and fluxes within a forest canopy, *J. Geophys. Res.*, *104*, 6081–6091, 1999.

Katul, G. G., and C.-I. Hsieh, A reply to the comment by Bink and Meesters, *Boundary Layer Meteorol.*, *84*, 503–509, 1997.

Katul, G. G., S. M. Goltz, C.-I. Hsieh, Y. Cheng, F. Mowry, and J. Sigmon, Estimation of surface heat and momentum fluxes using the flux variance method above uniform and non-uniform terrain, *Boundary Layer Meteorol.*, *74*, 237–260, 1995.

Katul, G. G., C.-I. Hsieh, R. Oren, D. Ellsworth, and N. Phillips, Latent and sensible heat flux predictions from a uniform pine forest using surface renewal and flux-variance methods, *Boundary Layer Meteorol.*, *80*, 249–282, 1996.

Katul, G. G., C. D. Geron, C.-I. Hsieh, B. Vidakovic, and A. B. Guenther, Active turbulence and scalar transport near the forest-atmosphere interface, *J. Appl. Meteorol.*, *37*, 1533–1546, 1998.

Katul, G. G., et al., Spatial variability of turbulent fluxes in the roughness sublayer of an even-aged forest, *Boundary Layer Meteorol.*, *93*, 1–28, 1999.

Kroon, L., and H. DeBruin, The Crau field experiment: Turbulent exchange in the surface layer under conditions of strong local advection, *J. Hydrol.*, *166*, 327–351, 1995.

Kustas, W., J. Blanford, D. Stannard, C. Daughtry, W. Nichols, and M. Weltz, Local energy flux estimates for unstable conditions using variance data in semiarid rangelands, *Water Resour. Res.*, *30*, 1351–1361, 1994.

Lai, C.-T., and G. G. Katul, The dynamic role of root-water uptake in coupling potential to actual transpiration, *Adv. Water Resour.*, *23*, 427–439, 2000.

Lai, C.-T., G. G. Katul, D. S. Ellsworth, and R. Oren, Modeling vegetation-atmosphere CO_2 exchange by a coupled Eulerian-Lagrangian approach, *Boundary Layer Meteorol.*, *95*, 91–122, 2000a.

Lai, C.-T., G. G. Katul, R. Oren, D. S. Ellsworth, and K. Schäfer, Modeling CO and water vapor turbulent flow distributions within a forest canopy, *J. Geophys. Res.*, *105*, 26,333–26,351, 2000b.

Lloyd, C. A., A. Culf, A. Dolman, and J. Gash, Estimates of sensible heat flux from observations of temperature fluctuations, *Boundary Layer Meteorol.*, *57*, 311–322, 1991.

Miller, K. S., and B. Ross, *An Introduction to the Fractional Calculus and Fractional Differential Equations*, 360 pp., John Wiley, New York, 1993.

Monin, A. S., and A. M. Obukhov, Basic laws of turbulent mixing in the ground layer of the atmosphere, *Tr. Geofiz. Inst. Akad. Nauk. SSSR*, *24*, 163–187, 1954.

Monji, N., Budgets of turbulent energy and temperature variance in the transition zone from forced to free convection, *J. Meteorol. Soc. Jpn.*, *51*, 133–145, 1973.

Ohtaki, E., On the similarity in atmospheric fluctuations of carbon dioxide, water vapor and temperature over vegetated fields, *Boundary Layer Meteorol.*, *32*, 25–37, 1985.

Oldham, K. B., and J. Spanier, *The Fractional Calculus*, 234 pp., Academic, San Diego, Calif., 1974.

Padro, J., An investigation of flux-variance methods and universal function applied to three land-use types in unstable conditions, *Boundary Layer Meteorol.*, *11*, 355–361, 1993.

Parlange, M. B., and W. Brutsaert, Regional roughness of the Landes Forest and surface shear stress under neutral conditions, *Boundary Layer Meteorol.*, *48*, 69–81, 1993.

Paw U, K. T., J. Qiu, H.-B. Su, and T. Watanabe, Surface renewal analysis, a new method to obtain scalar, *Agric. For. Meteorol.*, *74*, 119–137, 1995.

Priestly, C. H. B., *Turbulent Transfer in the Lower Atmosphere*, 130 pp., Univ. of Chicago Press, Chicago, Ill., 1959.

Raupach, M. R., J. J. Finnigan, and Y. Brunet, Coherent eddies and turbulence in vegetation canopies: The mixing-layer analogy, *Boundary Layer Meteorol.*, *78*, 351–382, 1996.

Spanier, J., Education in applied mathematics—The Claremont Mathematics Clinic, *SIAM Rev.*, *19*, 536–549, 1977.

Stull, R. B., *An Introduction to Boundary Layer Meteorology*, 666 pp., Kluwer Acad., Norwell, Mass., 1988.

Tillman, J., The indirect determination of stability, heat and momentum fluxes in the atmospheric boundary layer from simple scalar variables during dry unstable conditions, *J. Appl. Meteorol.*, *11*, 783–792, 1972.

Wang, J., and R. L. Bras, A new method for estimation of sensible heat flux from air temperature, *Water Resour. Res.*, *34*, 2281–2288, 1998.

Weaver, H. L., Temperature and humidity flux-variance relations determined by one-dimensional eddy correlation, *Boundary Layer Meteorol.*, *53*, 77–91, 1990.

Wesley, M., Use of variance techniques to measure dry air-surface exchange rates, *Boundary Layer Meteorol.*, *44*, 13–31, 1988.

Wyngaard, J. C., O. R. Coté, and Y. Izumi, Local free convection, similarity and the budgets of shear stress and heat flux, *J. Atmos. Sci.*, *28*, 1171–1182, 1971.

G. Katul, C.-T. Lai, and K. H. Wesson, School of the Environment, Duke University, Box 90328, Durham, NC 27708-0328. (gabrielk@env.duke.edu; cl9@duke.edu; khw7f@duke.edu)

(Received May 18, 2000; revised January 9, 2001; accepted January 12, 2001.)