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The Lagrangian stochastic model for fetch and latent heat flux estimation above uniform and nonuniform terrain

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Abstract. A Lagrangian stochastic model was used to estimate the fetch and latent heat flux above a nonuniform grass-covered forest clearing site at the Duke Forest, in Durham, North Carolina, and an irrigated bare soil patch at the University of California in Davis. The latent heat flux predictions by the Lagrangian model compared well with eddy correlation flux measurements. In order to apply the Lagrangian model to a nonuniform grass-covered forest clearing, the surface was treated as an imaginary "equivalent" uniform terrain subjected to surface roughness and turbulence statistics (i.e., mean, variance, and covariance) of velocities and scalars identical to those measured above the nonuniform terrain. At the irrigated bare soil site the equilibrium distance of the air from dry to wet was well defined, and its influence on the water vapor flux internal boundary layer was considered. In the Lagrangian model, five different schemes to account for inhomogeneous turbulent flows were compared in terms of estimating scalar fluxes. Our comparisons demonstrate that the five different schemes produce similar scalar fluxes despite the fact that some of them do not satisfy the well-mixed criterion. Also, the analytical solution to the advection-diffusion equation was used to predict the fetch and latent heat flux under neutral conditions and compared to the Lagrangian model.

1. Introduction

Understanding turbulent transport of water vapor from natural and vegetated surfaces continues to be an active research topic in surface hydrology and land-atmosphere interaction. More specifically, attention is devoted to measurements and modeling of water vapor fluxes in the surface layer under advective conditions for nonuniform terrain [e.g., *de Bruin et al.*, 1991; *Kustas et al.*, 1994; *Katul et al.*, 1995]. In order to understand the role of advective transport and terrain nonuniformity on water vapor fluxes, it is necessary to investigate the turbulence dispersion mechanisms within the local internal boundary layer (IBL) in equilibrium with the surface of interest.

Generally, an IBL is formed when air flows over (1) a surface transition, such as a sudden change in surface roughness, or (2) a surface which provides different sources of scalars, such as air flow from dry to wet surfaces. Such a classification was suggested by *de Bruin et al.* [1991]; in the first case, both scalar and momentum transport are influenced by the IBL, while in the second case, scalar transport is much more influenced by the IBL when compared to momentum transport.

Owing to the formation of this IBL, it is necessary to establish a fetch long enough to ensure that the depth of the IBL is greater than the height of measurement and that the flow attains an equilibrium state with the surface. The "fetch" is the downwind distance from the point that the surface feature changes. The "effective fetch" or source area can be interpreted as the integral of the source weight function (here source weight function is used instead of "footprint," as suggested by *Schmid and Oke* [1988] and *Schmid* [1994]). In the hydrologic and land-atmosphere interaction experiments, estimating the scalar fluxes and "fetch" or "source weight function" using Lagrangian stochastic models and Eulerian closure models has been the subject of recent research activities [e.g., *Luhar and Rao*, 1994; *Horst and Weil*, 1992; *Wilson and Swaters*, 1991; *Schuepp et al.*, 1990; *Leclerc and Thurtell*, 1990; *Gash*, 1986; *Horst and Slinn*, 1984].

Recently, *Luhar and Rao* [1994] used a Lagrangian model to investigate the "footprint" and they compared the water vapor fluxes calculated by a second-order Eulerian model and a Lagrangian model over a surface subjected to a sudden change in momentum roughness and moisture content. Their results show that the scalar flux calculations by the Eulerian and Lagrangian model are in good agreement. *Dolman and Wallace* [1991] also showed both the Lagrangian and Eulerian (K theory) models reproduce measured evaporation well when the near-field effect is small. However, using K theory to model the fluxes within the canopy is still questionable [e.g., *Denmead and Bradley*, 1985] owing to the near-field effect and counter-gradient transfer. As discussed by *Raupach* [1988], Lagrangian models are better suited for this purpose.

Leclerc and Thurtell [1990] first applied a Lagrangian model to estimate the source weight function and concluded that "the 100 to 1 fetch-to-height ratio grossly underestimates fetch re-

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quirements when observations are carried out above smooth surfaces, in stable conditions, or at high observation levels." While *Leclerc and Thurtell*'s [1990] results demonstrated the usefulness of Lagrangian stochastic models in predicting fetch requirements, uncertainty in incorporating the step changes in atmospheric thermal conditions and surface roughness still persist, and comparisons with field measurements are still scarce.

The difficulty in using Lagrangian models to estimate the scalar fluxes and fetch (or source weight function) is classically attributed to the inhomogeneity in the vertical velocity variance. Incorporating such inhomogeneity is not unique and motivated the development of many Lagrangian stochastic model corrections [see, e.g., *Baldocchi*, 1992, for a review]. In this study, five different model correction schemes are compared for estimating scalar fluxes. The field testing of the Lagrangian model is carried out using (1) a "forward" approach where given the source area, the Lagrangian model is used to predict latent heat fluxes that are compared with eddy correlation measurements, and (2) a "backward" approach where given a partial description of the flux profile from eddy correlation measurements, the Lagrangian model is applied to predict the fetch and source weight function.

The specific objectives include (1) comparing five schemes of incorporating the turbulent inhomogeneity in the Lagrangian model for estimating scalar fluxes; (2) investigating the feasibility of using such a Lagrangian stochastic approach to predict fetch and water vapor fluxes above nonuniform terrain such as a forest clearing and advective conditions such as an irrigated bare soil patch; and (3) comparing the Lagrangian model predictions with the analytical solution to the advection-diffusion equation and eddy correlation measurements.

2. Theory

This section is divided into two parts. In the first part the five different schemes for correcting the vertical inhomogeneity in the Lagrangian models are presented, and in the second part the analytical solution to the advection-diffusion equation proposed by *Gash* [1986] for neutral conditions and uniform wind field is summarized.

2.1. The Lagrangian Stochastic Models

While the Eulerian momentum conservation equations for viscous fluids (Navier-Stokes equations) can be transformed into the Lagrangian frame of reference [see *Monin and Yaglom*, 1971, pp. 531–532], the result of this transformation is not very convenient from a computational point of view since the viscous interaction is described by nonlinear terms of the fifth degree. Instead, phenomenological models are used to describe the Lagrangian velocity field.

Lagrangian stochastic models for vertical dispersion are based on the assumption that the vertical velocity of a marked material particle (air parcel) can be defined, in analogy to a diffusion process, by the drift and random acceleration terms. That is, to a first approximation, the vertical velocity of an air parcel is described by a Langevin-type stochastic differential equation [see *Gardiner*, 1990, pp. 80–116; *Durbin*, 1983; *Thomson*, 1987; *Legg and Raupach*, 1982; *Wilson and Sawford*, 1996]. In this study an air parcel is defined as a tiny connected lump of air containing many water vapor molecules that is smaller in size than the Kolmogorov dissipation length scale [*Tennekes and Lumley*, 1972, p. 20]. It is also assumed that this parcel is a passive substance and remains connected throughout its motion without being distorted by the turbulent flow.

This approximation to the vertical velocity, in finite difference form, leads to a Markov chain describing the vertical velocity (w) time series as

$$w_{i+1} = \alpha w_i + \beta \sigma_w(z_{i+1}) \eta_{i+1} \tag{1}$$

where $\alpha \approx \exp(-\Delta t/t_L)$, Δt is the time step, t_L is the Lagrangian timescale (by choosing $\Delta t = 0.1 t_L$, α becomes a constant), $\beta = (1 - \alpha^2)^{1/2}$, σ_w is the standard deviation of w at a fixed point, and η is a Gaussian random variable with zero mean and unit variance. The general derivation of (1) is given by *Hall* [1975], *Wilson et al*. [1981a], *Legg and Raupach* [1982], *Ley* [1982], *Durbin* [1983], *Legg* [1983], and *Sawford* [1985].

However, (1) is not suitable for inhomogeneous turbulent flows since σ_w is not constant with height. For the unstable atmospheric surface layer, σ_w increases with height; thus particles released near the ground surface are trapped in a region which has a small vertical dispersion and will have a low probability of leaving this region. This reduction in particle dispersion leads to an undesired particle accumulation, which occurs when (1) is used to compute particle trajectories. As shown by *Thomson* [1987], this accumulation leads to the violation of the "well-mixed criterion" that must be satisfied for stochastic models of particle trajectories. The well-mixed criterion states that if parcels of air carrying water vapor are initially well mixed, they must remain so throughout the dispersion process. To prevent this artificial mass accumulation, a drift correction is generally added to (1). Here we consider the five schemes for this drift correction term, proposed by *Wilson et al*. [1981b, 1983], *Legg and Raupach* [1982], *Thomson* [1987], and *Leclerc et al*. [1988]. While the rationale for the correction term is the same, the implementation of the drift term in (1) is distinct in these studies, and thus all five corrections will be considered in our study. It should be noted that the so called "random flight" models have been proposed to account for the inhomogeneity and non-Gaussian turbulence [see, e.g., *Sawford*, 1986, 1993; *Luhar and Britter*, 1989]. However, as noted by *Raupach* [1988], random flight models suffer from mathematical problems in strongly non-Gaussian turbulence and the calculations are lengthy and noisy owing to the large number of particles and time steps required. Thus, from an operational perspective, these models will not be considered in this study.

1. The *Wilson et al*. [1981b] (WTK) and *Wilson et al*. [1983] (WTK″) models: The WTK model states that for an inhomogeneous turbulent flow, the vertical velocity of a particle can be expressed as

$$q_{i+1} = \alpha q_i + \beta \eta_{i+1} \tag{2a}$$

$$w_i = \sigma_w(z_i) q_i + t_L(z_i) \sigma_w(z_i) \left. \frac{\partial \sigma_w}{\partial z} \right|_{z=z_i} \tag{2b}$$

where q_i is a dimensionless vertical velocity time series, and the second term on the right-hand side of (2b) is the drift correction term that depends on the level of inhomogeneity. Under the assumption that the gradient of σ_w changes gradually with height, *Wilson et al*. [1983] rewrote WTK as WTK″, which expresses the vertical velocity of a particle as

$$S_{i+1} = \alpha S_i + \beta \eta_{i+1} + \gamma t_L(z_{i+1}) \left. \frac{\partial \sigma_w}{\partial z} \right|_{z=z_{i+1}} \tag{3a}$$

Table 1. Summary of the Five Lagrangian Models Used

Model	Equation(s)	Comment
Wilson et al. [1981b] (WTK)	$q_{i+1} = \alpha q_i + \beta\eta_{i+1}$ $w_i = \sigma_w(z_i)q_i + t_L(z_i)\sigma_w(z_i)\ (\partial\sigma_w/\partial z)\|_{z=z_i}$	Markov chain is expressed in terms of w/σ_w
Wilson et al. [1983] (WTK″)	$S_{i+1} = \alpha S_i + \beta\eta_{i+1} + \gamma t_L(z_{i+1})\ (\partial\sigma_w/\partial z)\|_{z=z_{i+1}}$ $w_i = S_i\sigma_w(z_i)$	Markov chain is expressed in terms of w/σ_w
Legg and Raupach [1982] (LR)	$w_{i+1} = \alpha w_i + \beta\sigma_w(z_{i+1})\eta_{i+1}$ $+ 2\gamma t_L(z_{i+1})\sigma_w\ (\partial\sigma_w/\partial z)\|_{z=z_{i+1}}$	Markov chain is expressed in terms of w
Thomson [1987]	$w_{i+1} = \alpha w_i + \beta\sigma_w(z_{i+1})\eta_{i+1} + 2\ \gamma t_L(z_{i+1})\sigma_w(z_{i+1})$ $\cdot\frac{1}{2}[(w_i^2/\sigma_w^2(z_i)) + 1](\partial\sigma_w/\partial z)\|_{z=z_{i+1}}$	Markov chain is expressed in terms of w
Leclerc et al. [1988] (LTK)	$S_{i+1} = \alpha S_i + \beta\eta_{i+1}$ $w_i = S_i\sigma_w(z_i)$ $\Delta z_{i+1} = \Delta z_i + w_i\Delta t$ + probability of particle reflection	Markov chain is expressed in terms of w/σ_w

$$w_i = S_i\sigma_w(z_i) \tag{3b}$$

where S_i is a dimensionless time series and γ $(= 1 - \alpha)$ is a constant. In both the WTK and WTK″ models the instantaneous vertical velocity (w_i) is thought of as the dimensionless time series (q_i or S_i) multiplied by the standard deviation of vertical velocity (σ_w).

2. The *Legg and Raupach* [1982] (LR) model: The LR model for w is given by

$$w_{i+1} = \alpha w_i + \beta\sigma_w(z_{i+1})\eta_{i+1} + 2\gamma t_L(z_{i+1})\sigma_w(z_{i+1})\left.\frac{\partial\sigma_w}{\partial z}\right|_{z=z_{i+1}} \tag{4}$$

where the third term on the right-hand side of (4) is considered as the drift correction term. The relations between the WTK, WTK″, and LR models are further discussed by *Wilson et al.* [1983].

3. The *Thomson* [1987] model: Thomson's model describes the Markov chain for the vertical velocity as

$$w_{i+1} = \alpha w_i + \beta\sigma_w(z_{i+1})\eta_{i+1} + 2\gamma t_L(z_{i+1})\sigma_w(z_{i+1}) \cdot\left(\frac{1}{2}\left(\frac{w_i^2}{\sigma_w^2(z_i)} + 1\right)\right)\left.\frac{\partial\sigma_w}{\partial z}\right|_{z=z_{i+1}}. \tag{5}$$

Notice that if w_i^2 is identical to σ_w^2, Thomson's model reduces to the LR model.

4. The *Leclerc et al.* [1988] (LTK) model: Instead of adding a drift term directly to the Markov chain in (1), the LTK model suggests a novel approach that utilizes a particle reflection scheme to eliminate the artificial mass accumulation for inhomogeneous turbulent flows. In essence, the reflection scheme still relies on the degree of inhomogeneity in the vertical velocity variance. Particles have a finite probability of being reflected upward when $\partial\sigma_w/\partial z > 0$ or downward when $\partial\sigma_w/\partial z < 0$. A uniformly distributed random variable (r_x), which has zero mean and unit variance, is used to determine the probability of particle reflection. If r_x is greater than the reflection ratio p $(= \sigma_w(z_{i+1})/\sigma_w(z_i))$, then a particle reflection will occur [see *Leclerc et al.*, 1988].

We summarize these five models in Table 1. The implementation of the models is considered next.

Neglecting the longitudinal velocity fluctuations (i.e., longitudinal dispersion), a two-dimensional particle trajectory can be expressed as

$$x_{i+1} = x_i + \Delta x \tag{6a}$$

$$z_{i+1} = z_i + \Delta z \tag{6b}$$

$$\Delta x = U_i\Delta t \tag{6c}$$

$$\Delta z = w_i\Delta t \tag{6d}$$

where x_i and z_i are the downwind longitudinal and vertical positions of the particle, respectively, U_i is the mean longitudinal wind velocity at t_i, and w_i can be described by the five models mentioned above. Hence the Eulerian velocity field has to be specified or estimated a priori in order to drive the Lagrangian stochastic model to predict the particle trajectory. The estimation of the Eulerian velocity statistics and calculation of cumulative flux are considered next.

1. Horizontal mean wind profile: Above the canopy the mean wind profile can be estimated using Monin-Obukhov surface layer similarity theory [see also *Brutsaert*, 1982, pp. 66–70]:

$$U(z) = \frac{u_*}{k}\left[\ln\left(\frac{z-d}{z_0}\right) - \psi_m\left(\frac{z}{L}\right)\right] \tag{7}$$

where u_* is the friction velocity, z_0 is the surface roughness length, d is the zero plane displacement height, L is the Obukhov length, and ψ_m is defined as

$$\psi_m\left(\frac{z}{L}\right) = \ln\left[\frac{(1+\chi)^2(1+\chi^2)}{(1+\chi_0)^2(1+\chi_0^2)}\right] - 2\arctan(\chi) + 2\arctan(\chi_0) \qquad z/L < 0 \tag{8a}$$

$$\psi_m\left(\frac{z}{L}\right) = -5\left[\left(\frac{z-d}{L}\right) - \left(\frac{z_0}{L}\right)\right] \qquad z/L > 0 \tag{8b}$$

where $\chi = \{1 - 16[(z-d)/L]\}^{1/4}$ and $\chi_0 = [1 - 16(z_0/L)]^{1/4}$. For free-convective conditions (8a) is not adequate beyond $-(z-d)/L > 3$ as evidenced by *Kader and Yaglom* [1990] and *Parlange and Katul* [1995].

Within the canopy, $U(z)$ is taken as [*Legg and Raupach*, 1982]

$$U(z) = U(h)\exp\left(3.0\left(\frac{z}{h} - 1\right)\right) \tag{9}$$

where h is the height of the canopy, $U(h)$ is the mean longitudinal velocity at the canopy top and is calculated from (7) by setting $z = h$.

2. The σ_w profile: Above the canopy, σ_w can be estimated by [*Panofsky and Dutton*, 1984, p. 161]

$$\sigma_w = 1.25u_*\left(1 - 3\left(\frac{z-d}{L}\right)\right)^{1/3} \qquad z/L < 0 \tag{10a}$$

$$\sigma_w = 1.25u_* \qquad z/L \geq 0. \tag{10b}$$

Within the canopy, σ_w is assumed to decrease linearly with height. A suggested relation by *Wilson* [1980]

$$\sigma_w(z) = \left(\frac{\sigma_w(h) - 0.1\sigma_w(h)}{h}\right)z + 0.1\sigma_w(h) \tag{11}$$

is used in this study, where $\sigma_w(h)$ is the standard deviation of vertical velocity at the canopy top calculated from (10) for $z = h$. The σ_w value at the ground is taken as $0.1\sigma_w(h)$.

3. The t_L profile: Above the canopy, the Lagrangian timescale is taken as

$$t_L = K/\sigma_w^2 \tag{12a}$$

$$K = kzu_*/\phi \tag{12b}$$

to match the dispersion given by K theory, where K is the turbulent eddy diffusivity and k (= 0.4) is von Karman's constant (for detailed discussion see work by *Taylor* [1921], *Csanady* [1973], *Angell* [1974], *Reid* [1979], and *Ley and Thomson* [1983]). The stability function ϕ is taken as ϕ_h by *Monin and Yaglom*'s [1965] suggestion that the eddy diffusivities for heat and a passive substance are equal. Here ϕ_h is estimated by (see Appendix A)

$$\phi_h = 0.32[0.037 - (z-d)/L]^{-1/3} \qquad z/L < 0 \tag{13a}$$

$$\phi_h = 1 + 5(z-d)/L. \qquad z/L > 0 \tag{13b}$$

It should be noted that the Lagrangian timescale in (12a) is valid for far enough travel time from the source (far field). Close to the ground surface (near field), (12a) is not accurate since the source variability becomes significant [*Corrsin*, 1974; *Raupach*, 1988, 1989a, b].

Within the canopy, as a first approximation, t_L is assumed to be a constant [*Leclerc*, 1987; *Legg and Raupach*, 1982] and has the same value as that at the canopy top. Data presented by *Raupach* [1988] suggest that such an approximation is reasonable.

4. Relative cumulative flux: Particles are released at the canopy top ($z = h$) from an infinite cross-wind line source, the travel times are chosen to reach the constant flux layer, and in order to achieve stable statistical results, 3000 particles are released for a single simulation. *Sawford* [1985] suggested 5000 particles to be released. We have carried out a sensitivity analysis on the number of particles, and we did not find significant differences between releasing 3000 or 5000 particles in our models. In this study, z_0 is set to be a perfect reflection boundary. Particles are perfectly reflected when they reach or cross this boundary.

To calculate the flux at a certain height z, and downwind distance x, the number of particles must be counted at (x, z). The relative flux $F(x, z)$ is expressed as

$$F(x, z) = (n_\uparrow - n_\downarrow)/N,$$

where $n_\uparrow$ and $n_\downarrow$ are the number of particles which cross the height z at x with the upward and downward direction, respectively, and N is the number of particles released. Hence the relative cumulative flux $\mathrm{CF}(x, z)$ can be expressed as

$$\mathrm{CF}(x, z) = \int_0^x (F(x, z)/dx)\, dx.$$

Now the source strength for water vapor flux was defined by

$$\mathrm{SS} = \frac{\mathrm{LE}_m(x, z)}{\mathrm{CF}(x, z)},$$

where SS is the source strength and LE_m is the latent heat flux measured by the eddy-correlation system. For an infinite down wind distance (fetch) the relative cumulative flux approaches unity.

2.2. Analytical Solution

Gash [1986] used *Calder*'s [1952] solution of the advection-diffusion equation for a neutral atmosphere and a uniform wind field U_u to derive

$$x_F = -\frac{U_u z_m}{ku_*}\frac{1}{\ln(F_p/100)} \tag{14}$$

where z_m is the measurement height, F_p is the percentage of the scalar flux, x_F is the fetch requirement to achieve the F_p percent scalar flux, and U_u is the uniform wind velocity defined as

$$U_u = \frac{\int_{z_0}^{z_m} U\, dz}{\int_{z_0}^{z_m} dz}. \tag{15}$$

Using Gash's analysis, *Shuepp et al.* [1990] derived

$$\frac{d\,\mathrm{CF}(x, z)}{dx} = \frac{U_u(z-d)}{u_* kx^2}\exp\left[-U_u(z-d)/ku_* x\right]. \tag{16}$$

Integrating (16) from 0 to x, CF is given by

$$\mathrm{CF}(x, z) = \exp[-U_u(z-d)/ku_* x]. \tag{17}$$

Here, given the surface roughness length, (7), (15), and (17) are used to calculate the cumulative flux as a function of fetch analytically for neutral atmospheric conditions.

3. Experiments

The eddy correlation latent heat flux measurements were carried out (1) above a nonuniform grass-covered forest clearing at the Blackwood division of the Duke Forest, in Durham, North Carolina (on July 11 and October 23, 1994, at two separate locations), and (2) above a uniformly irrigated bare soil patch within a larger dry soil field at the Campbell Tract facility at the University of California in Davis.

3.1. Duke Forest Clearing Site

At this site, two experiments were carried under different soil wetness conditions, grass height, and instrumentation height.

3.1.1. Experiment 1 (October 23, 1994). The grass-covered site is approximately 480 by 305 m and is surrounded by Loblolly pine trees 10–13 m tall (the site elevation is 163 m). On October 23, a mast, situated 300 m from the north end and

250 m from the west end of the forest edge was constructed. Eddy correlation systems were located at 1.0, 1.5, and 2.6 m above the ground surface. At each height a Krypton hygrometer and CA27 one-dimensional sonic anemometer (Campbell Scientific) were used to measure the latent heat flux. A Gill triaxial sonic anemometer was also set at 2.75 m above the ground to measure the three velocity components and air temperature, and these measurements were used to calculate u_* and L.

The absolute air temperature was determined from the speed of sound (C_s) using $(C_s)^2 = c_r R_d T$, where $R_d (= 287.04$ J Kg^{-1} $\text{K}^{-1})$ is the gas constant of dry air at constant pressure and c_r ($= 1.4$) is the ratio of the molar specific heat capacities of air at constant pressure to that at constant volume. The adequacy of the Gill sonic anemometer to measure temperature fluctuations are discussed by *Katul* [1994] and *Katul et al.* [1994a]. The sampling frequency and duration for all measured variables (i.e., temperature, humidity, and wind velocity) were 10 Hz and 27 min, respectively, resulting in 16,384 data points per flow variable. During the experiment, the sky was clear and the wind direction varied from 200° to 300° (SSW through WNW).

The surface roughness length z_0 was calculated from (7) using the measured u_* and U under near-neutral conditions. We also neglected the zero-plane displacement height for this surface. Owing to the grass height variability, microtopography and large variability in wind direction, z_0 was found to be a function of wind direction and could be expressed empirically as $z_0 = ae^{bDw}$, where $a = 0.00257$, $b = 0.014$, and D_w is the mean wind direction (see Figure 1). The average canopy height (h) along the mean wind direction was $4z_0$. Hence h varied from 0.2 to 0.7 m depending on the wind direction.

In order to compare the analytical and the numerical solution to the field measurements, we selected the data that exhibit the following characteristics: (1) the atmosphere was near neutral, (2) there was a measurable decrease in the latent heat flux as a function of height, and (3) the latent heat flux at 2.6 m was at least 50 W m^{-2}. Owing to these stringent criteria for selecting data, only five runs are presented in this experiment.

3.1.2. Experiment 2 (July 11, 1994). On July 11 the mast was located at 50 and 100 m from the north end and west end portions of the forest edge, respectively. The eddy correlation systems were set at 0.88 and 2.0 m, respectively. Each eddy correlation system consisted of a Krypton hygrometer and a CA27 one-dimensional sonic anemometer to measure the latent heat flux. The three wind velocity components and air temperature were measured 1.54 m above the ground surface using a Gill triaxial sonic anemometer. Sampling frequency and duration were the same as those in experiment 1. Before this experiment was carried out, a rain event saturated the top 5 cm of the grass surface. After the rain event the sky was cloud-free.

In this experiment the measured mean canopy height was 23 cm. By the same methodology used in experiment 1, the surface roughness length was found to be 10 cm assuming the zero-plane displacement height is zero (and the variation of z_0 with wind direction was negligible for this experiment). We note that the grass was cut in mid-September.

We also carried out experiments to compare the Gill three-dimensional and Campbell CA27 one-dimensional sonic anemometers before we started these two experiments above the grass site. The comparison, shown in Appendix D, demonstrates that the two anemometers agree well for the purpose of this study.

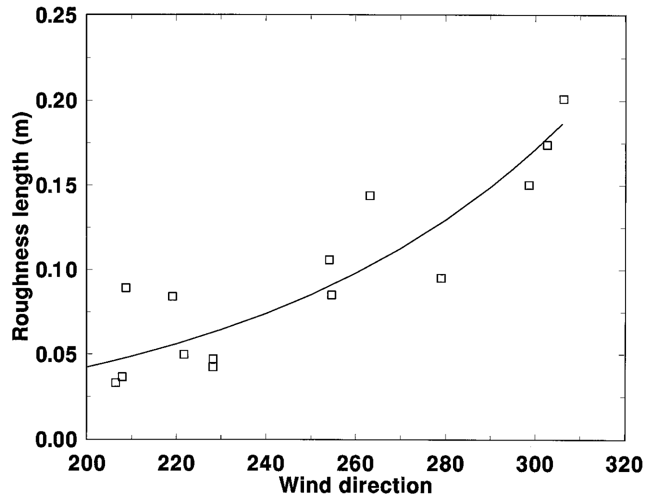

Figure 1. The roughness length z_0 is shown as a function of mean wind direction, D_w. The solid line represents $z_0 = ae^{bDw}$, where $a = 0.00257$ and $b = 0.014$.

3.2. Davis Campbell Tract Facility (Bare Soil Site)

The Campbell tract facility is a 500-by-500-m bare soil site ($z_0 = 1.7$ mm, determined by the same method used in experiment 1 at the Duke site) at the University of California in Davis (elevation = 16 m). The site is equipped with a sprinkler irrigation system that is capable of irrigating a 120-by-110-m soil patch with uniformity coefficients of about 85% (see work by *Katul and Parlange* [1992] and *Parlange et al.* [1992] for details on the irrigation system). The tower was installed towards the north end of the field. An irrigation was carried out on the night of August 21, 1993, and only the data collected after the irrigation were used. The irrigated distance along the mean longitudinal wind direction varied from 80 to 100 m with the wind directions. Two CA27 one-dimensional sonic anemometers and Krypton hygrometers were used to measure the vertical velocity and latent heat flux at 1.12 and 2.9 m above the ground surface. A Gill triaxial sonic anemometer was used to measure the wind velocity statistics and sensible heat flux at 1.96 m. The sampling frequency was 21 Hz, and the sampling period was 26 min, resulting in 32,768 data points from each measured flow variable. Further details about the experiment are given by *Katul et al.* [1994b].

4. Results and Discussion

To compare the five schemes and examine the applicability of the Lagrangian model to estimate the fetch and latent heat flux, this section is divided into four parts. In part 1, comparisons between the five models are presented. In part 2 the estimation of the fetch and latent heat flux above the nonuniform terrain at the Duke Forest site and the comparison with eddy correlation measurements are presented. In part 3 a discussion on the estimation of the source weight function for this nonuniform surface is provided. In part 4 the use of the Lagrangian model to predict latent heat fluxes under advective conditions is evaluated at the irrigated bare soil site. The Lagrangian model was tested by the "backward" approach in

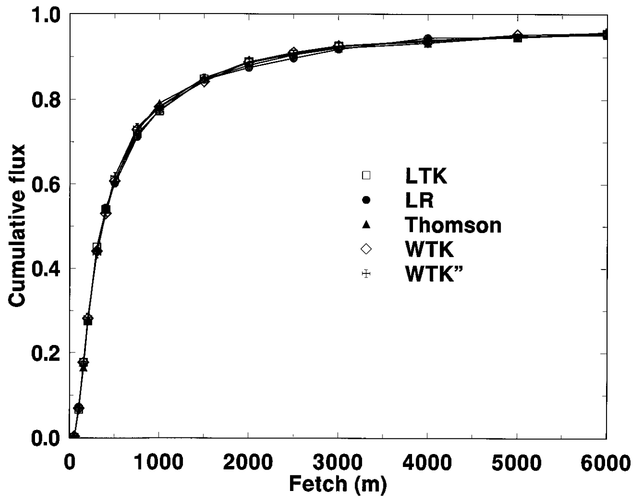

Figure 2a. Comparison between the five models to estimate the cumulative flux as a function of fetch under neutral conditions, where $u_* = 0.4$ m/s, $z_0 = 0.06$ m, $d = 0.3$ m, $h = 0.45$ m, and the observation height is 11.0 m. These conditions are from *Leclerc and Thurtell* [1990, Figure 10].

parts 2 and 3 and by the "forward" approach in part 4, as mentioned in the Introduction.

4.1. Comparison of the Five Models

If the assumptions that $\partial\sigma_w/\partial z$ changes only slowly with height and $w_i^2 = \sigma_w^2$ are valid, then WTK, WTK″, LR, and *Thomson*'s [1987] models are virtually the same (see Appendix B). Figures 2a–2c display the predicted cumulative flux using the five models as a function of fetch for neutral, stable, and unstable atmospheric stability conditions, respectively. In order to compare our simulations with *Leclerc and Thurtell*'s [1990] simulation, the surface and atmospheric conditions are chosen as those in Figure 10 of *Leclerc and Thurtell* [1990]. It is apparent that under unstable conditions (Figure 2c), the five models result in similar scalar flux estimations. However, it is important to note that the LR and LTK models do not satisfy the well-mixed criterion (see work by *Baldocchi* [1992] and Appendix C). Also, we note that for unstable conditions, the height variation in σ_w is most pronounced when compared to neutral or stable conditions, and thus the differences among these schemes must be maximum.

In estimating scalar fluxes, *Baldocchi* [1992] reported that the LR and *Thomson*'s [1987] models yield identical results (which is consistent with Figure 2c); however, he also found that the LTK reflection scheme does not prevent accumulation of particles near the ground. This interesting discrepancy between our findings in Figure 2c and those of *Baldocchi* [1992] is examined in Appendix C. From Figures 2a–2c it is apparent that the fetch requirements are strongly dependent on the stability conditions. Under unstable conditions the fetch requirement is shorter than that for neutral conditions and much shorter than that for stable conditions; these results are similar to *Leclerc and Thurtell*'s [1990]. Since these five models yield similar results, we decided to use *Thomson*'s [1987] model for latent heat flux predictions throughout the remainder of this study. *Thomson*'s [1987] model is derived directly from the well-mixed criterion.

4.2. Estimation of Latent Heat Fluxes at the Duke Forest Clearing (Backward Approach)

To implement the Lagrangian model for estimating the fetch and latent heat flux, the following two assumptions are necessary:

1. Since the experimental site is nonuniform at scales smaller than the IBL fetch, we define an "equivalent" uniform terrain subjected to identical surface roughness and turbulence statistics (i.e., mean, variance, and covariance) of velocities and scalars as those measured over the nonuniform terrain. That is, in order to apply (7)–(13), a planar homogeneous velocity field is assumed.
2. The ergodic hypothesis is valid in the Lagrangian frame of reference [see *Monin and Yaglom*, 1971, pp. 214–218; *Fischer et al.*, 1979, pp. 60–62] so that ensemble of particles released from a source can be replaced by many individual particle releases.

In experiment 1, at the Duke Forest clearing, eddy correlation measured latent heat flux (LE) at three different heights were available ($z = 1.0$ m, $z = 1.5$ m, and $z = 2.6$ m). Hence the "backward" approach, where a measured partial descrip-

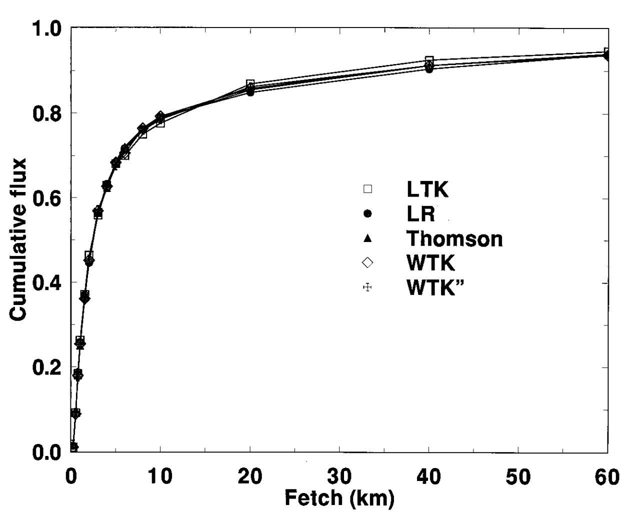

Figure 2b. Same as Figure 2a but for stable conditions ($L = +10$ m).

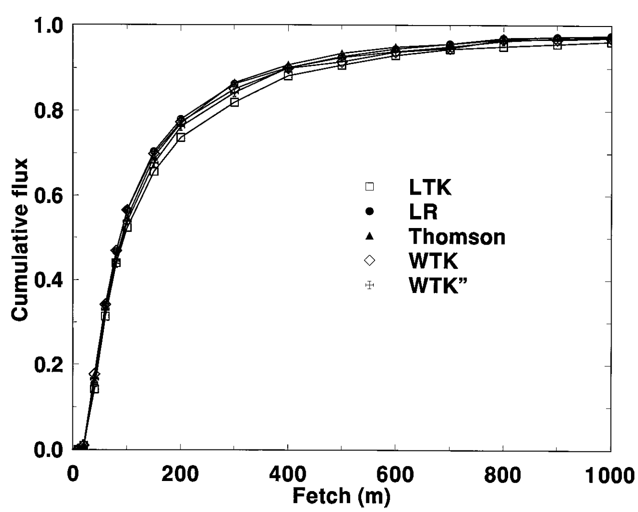

Figure 2c. Same as Figure 2a but for unstable conditions ($L = -10$ m).

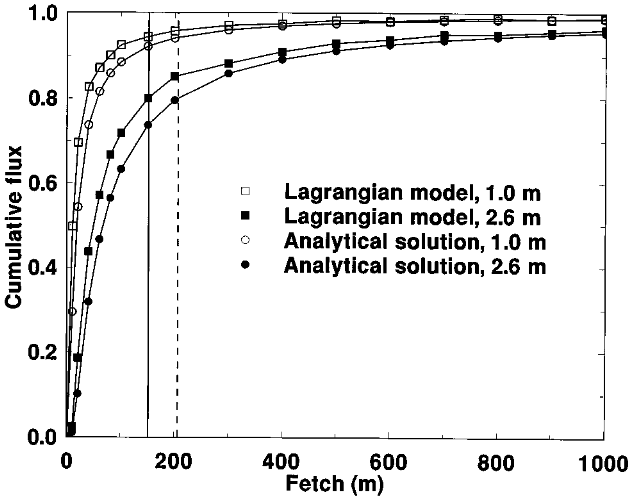

Figure 3. Comparison between the Lagrangian model [*Thomson*, 1987] and analytical solution–predicted fetch requirements for the imaginary uniform terrain, where $L = -1694.88$ m, $u_* = 0.25$ m/s, $z_0 = 0.063$ m, $d = 0.0$ m, $h = 0.25$ m, and observation heights are 1.0 and 2.6 m, respectively. The solid and dashed vertical lines represent the fetch requirements predicted by the Lagrangian model and analytical solution, respectively, that match the eddy correlation measurements.

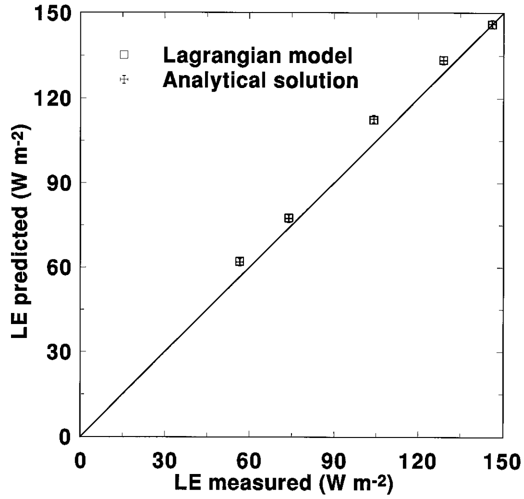

Figure 4. Comparison between the eddy correlation–measured, analytical solution, and Lagrangian model [*Thomson*, 1987] predicted LE. The 1:1 line is also shown.

tion of the latent heat flux profile is given, is applied to predict the fetch. Here, we used the measured latent heat flux ratio of 2.6 to 1.0 m to estimate the fetch requirement. To confirm this fetch estimation, we used this computed fetch and the Lagrangian model to calculate the latent heat flux at 1.5 m. The Lagrangian model predicted and eddy correlation measured latent heat fluxes at 1.5 m are then compared.

The comparison between Lagrangian model predictions and eddy correlation measurements was carried out by the following steps:

1. The latent heat flux ratio of 2.6 to 1.0 m was determined using the eddy correlation measured values.

2. From measured u_* and L, *Thomson*'s [1987] model was used to estimate the relative cumulative flux at 1.0, 1.5, and 2.6 m as a function of fetch.

3. The results from the second step were used to plot the cumulative flux ratio of 2.6 to 1.0 m as a function of fetch. Now, the fetch distance that results in a cumulative flux ratio identical to that of step 1 can be identified. This fetch distance is the estimated fetch by the model.

4. To confirm this fetch estimation, the cumulative flux curve for $z = 1.5$ m from step 2 was used with the estimated fetch from step 3 to determine the latent heat flux at 1.5 m. This estimated latent heat flux at $z = 1.5$ m was then compared with the eddy correlation measurements.

The procedure of using the analytical solution [*Gash*, 1986] to predict the fetch and LE is the same as that for the Lagrangian model, but we use the analytical solution to compute the cumulative flux. In Figure 3 a typical cumulative flux comparison between the Lagrangian model [*Thomson*, 1987] and the analytical solution described by (15) and (17) is shown. The solid and dashed vertical lines represent the estimated fetch predicted by the Lagrangian model and the analytical solution, respectively. The estimated fetch by the Lagrangian model and analytical solution for the five runs in experiment 1 are compared and summarized in Table 2. The fetch difference between the Lagrangian model and analytical solution estimations in Table 2 may be attributed to the fact that the atmosphere is not fully neutral and, more importantly, the assumption of a uniform wind velocity profile (equation (15)) is not valid. We have no evidence to prove which prediction is more realistic; however, the Lagrangian model works well in unstable conditions, as discussed in section 4.4. These two methods provide the same order-of-magnitude fetch predictions, and the differences between the two predictions do not result in different LE predictions at 1.5 m, as evidence in Figure 4.

Table 2. Summary of the Estimated Fetch by the Lagrangian Model and Analytical Solution

					Fetch, m	
Run	z_0, m	h, m	u_*, m/s	L, m	Lagrangian Model*	Analytical Solution
1	0.061	0.244	0.236	−43.2	70.8	141.0
2	0.063	0.251	0.250	−1694.9	151.7	206.2
3	0.168	0.672	0.371	−133.4	120.2	194.3
4	0.178	0.711	0.343	−553.7	136.8	218.6
5	0.153	0.613	0.317	−391.9	136.2	219.8

**Thomson* [1987].

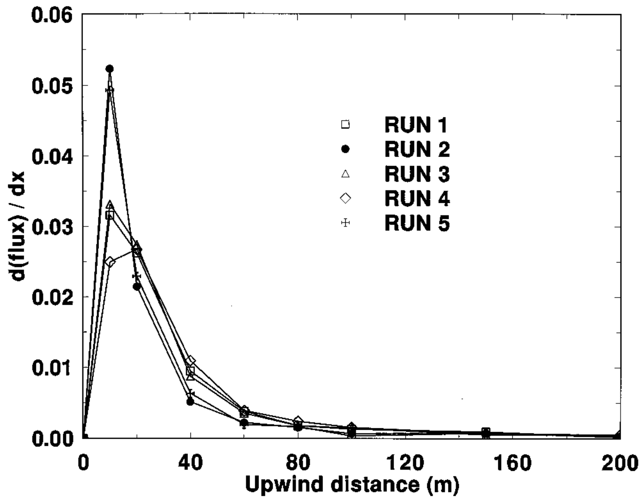

Figure 5. The longitudinal variation of the source weight functions predicted by the Lagrangian model [*Thomson*, 1987] as a function of upwind distance for five selected runs (u_*, 0.08–0.39 m/s; L, −5.91 *to* −19.16 m).

Figure 4 shows the comparison between the Lagrangian model and analytical solution predicted and the eddy correlation measured LE at $z = 1.5$ m. The Obukhov length (L) varied from −43.2 to −1694.9 m for the five runs in Figure 4. The good agreement noted in Figure 4 confirms the usefulness of Lagrangian model and analytical solution in providing order-of-magnitude fetch prediction and calculating LE profiles.

4.3. Source Weight Function at the Duke Forest Clearing (Backward Approach)

In experiment 2, at the Duke Forest clearing, the latent heat flux measurements were only available at two heights rather than three, as in experiment 1. The fetch can be predicted by the method described above. However, we do not have another LE measurement for independent verification of these predictions, as was the case in experiment 1. Rather than consider the fetch requirement, we investigated the usefulness of the Lagrangian model to predict the source weight function, $d(\mathrm{CF})/dx$, and qualitatively compare the outcome with expected field condition. Figure 5 shows the source weight functions predicted by *Thomson*'s [1987] model as a function of upwind distance for five different wind speeds (u_*) (0.08–0.39 m/s) and thermal stability conditions (L) (−5.91 to −19.16 m).

Figure 5 demonstrates that the main source area contributing to the latent heat flux at 2 m (the upper sensor in this experiment) is within 40 m from the mast. The proximity of the main contributing source to the measurement location implies that the latent heat fluxes measured at 0.88 and 2.0 m should be very close (i.e., the ratio of the two latent heat fluxes should be close to unity). We noticed that this similarity in source area persisted for all 10 available runs (though, for clarity purposes, we show only 5 runs in Figure 5). For further comparisons we show in Figure 6 the predicted source weight functions at 0.88 and 2.0 m for run 1. Notice that the maximum source strength is at 10 m.

This similarity in source proximity was realistic for experiment 2 since a rain event saturated the grass uniformly the day before. Figure 7 shows the ratio of the two latent heat fluxes measured at 0.88 and 2 m in experiment 2 and suggests that the Lagrangian model predictions are in good agreement with eddy correlation measurements. This good agreement further supports the usefulness of using the Lagrangian model to estimate the source weight function.

4.4. Latent Heat Fluxes From the Irrigated Bare Soil Experiment (Forward Approach)

The Davis bare soil irrigation experiment offers a unique opportunity to investigate the use of the Lagrangian model to predict latent heat fluxes under advective conditions. For this site the surface roughness length is constant and the step change that induces the formation of an IBL is a uniform water vapor source area surrounded by a dry surface. Hence the source area (fetch) is well defined in this experiment. Here the "forward" approach, where the source area is given, is used to predict the latent heat fluxes. For the purpose of comparing

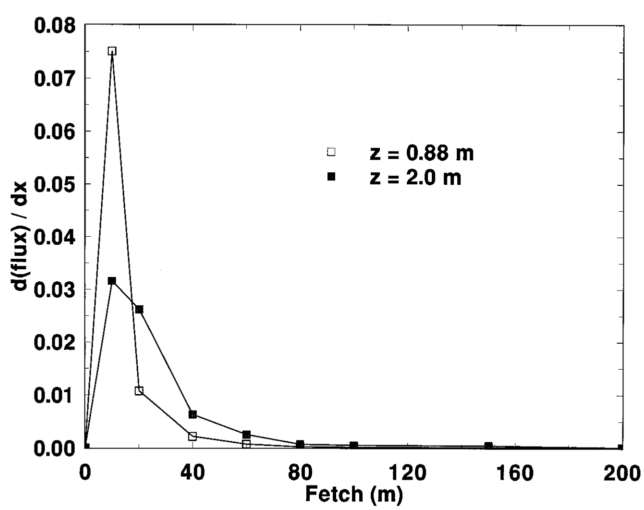

Figure 6. The variation of the source weight function as a function of fetch and height for run 1 ($L = -19.16$ m, $u_* = 0.387$ m/s, $z_0 = 0.1$ m, $d = 0.0$ m, $h = 0.23$ m, and observation heights are 0.88 and 2.0 m).

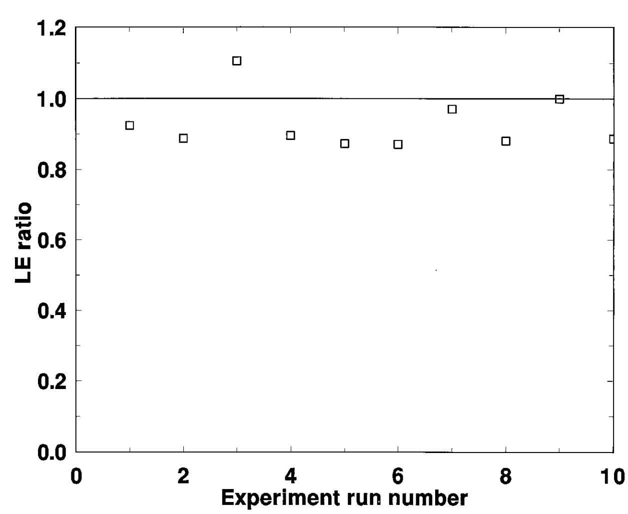

Figure 7. The ratio of the eddy correlation measured latent heat fluxes at 0.88 m and 2 m in Duke Forest experiment (2) following the rain event.

Lagrangian model predictions with measurements, the following procedure is used:

1. The relative cumulative latent heat fluxes at 1.12 and 2.9 m are predicted by *Thomson*'s [1987] model with $z_0 = 1.7$ mm, u_* and L measured from the Gill triaxial sonic anemometer, and a fixed fetch of 80–100 m (depends on the wind direction). The predicted LE ratio of 2.9 to 1.12 m is then computed.

2. Using the eddy correlation measurements, the measured LE ratio of 2.9 to 1.12 m is computed and compared to the predicted ratio in step 1. Here we compare the LE ratio rather than the absolute LE at 2.9 or 1.12 m. The advantage of doing this is that we do not have to define the source strength as described in the theory section.

The comparison between measured and predicted LE ratios for a wide range of atmospheric stability conditions is shown in Figure 8, in which we note the following: (1) for unstable atmospheric conditions, good agreement between the Lagrangian model predictions and eddy correlation measurements was found; (2) for stable atmospheric conditions the measurements were not very reliable (i.e., latent heat fluxes were less than 10 W m^{-2}), however, for completeness we show the comparison for stable conditions; and (3) for near-neutral conditions the Lagrangian model systematically underestimated the LE ratios when compared to the eddy correlation measurements.

To understand the systematic underestimation in near-neutral conditions, we consider the vertical velocity statistics at the location of the mast within the context of Monin-Obukhov similarity theory. Specifically, we tested (10a) at the three heights. If the three vertical velocity measurements are within the internal boundary layer and if the horizontal homogeneity in velocity statistics is achieved, then (10a) must describe the dimensionless variance (σ_w/u_*) with L measured at any height. In Figure 9 a comparison between eddy correlation measured and similarity theory predicted σ_w/u_* for all three heights as a function of $-z/L$ is shown, where L and u_* were measured at 1.96 m and σ_w was measured at 1.12, 1.96, and 2.9 m. Notice that measurements and prediction are in good agreement at 1.96 and 2.9 m; however, the measured values of σ_w/u_* at 1.12 m were below 1.25 (the minimum limit of prediction) for near-neutral conditions. This implies that u_* measured at 1.96 m cannot represent the u_* measured at 1.12 m, and it appears to be higher if neutral conditions prevail at 1.12 m. This also suggests that the atmospheric stability condition at 1.12 m should be stable. Hence the height of the internal boundary layer is between 1.12 and 1.96 m. The limited height of the IBL seems to explain the systematic underestimate of LE ratio by the Lagrangian model.

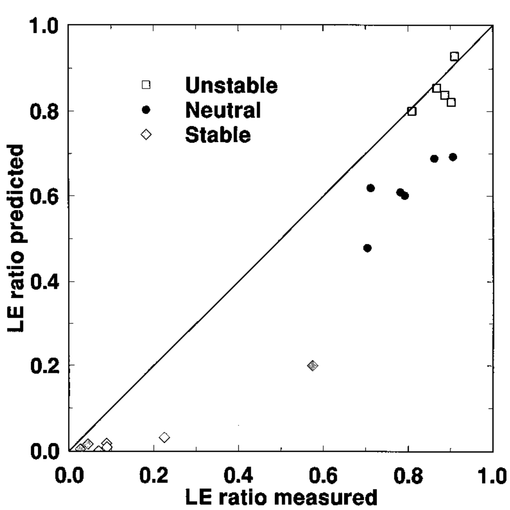

Figure 8. Comparison between predicted and measured latent heat flux ratio at 1.12 m and 2.9 m following an irrigation. The 1:1 line is also shown.

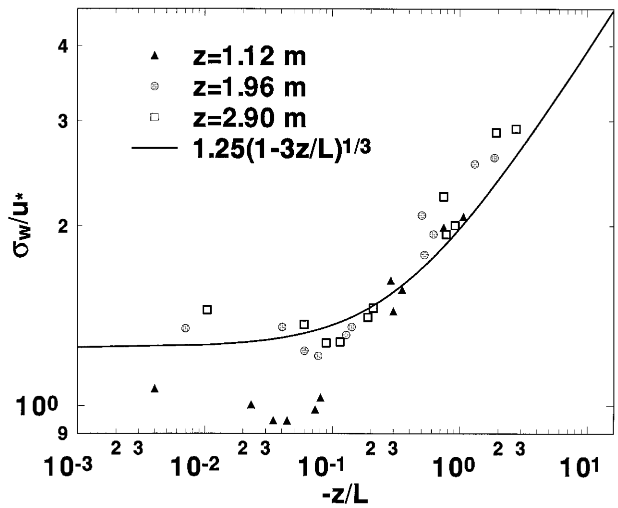

Figure 9. Comparison between eddy correlation–measured and similarity theory–predicted σ_w/u_* for all three heights as a function of $-z/L$, where L and u_* were measured at 1.96 m and σ_w was measured at 1.12, 1.96, and 2.9 m.

5. Conclusion

Five Lagrangian stochastic models, which differ in their drift correction terms for inhomogeneous turbulent flows, have been compared in estimating fluxes. *Thomson*'s [1987] model was used to estimate the fetch and latent heat flux above a nonuniform grass-covered forest clearing and a uniformly irrigated bare soil site. Our analysis demonstrated the following:

1. All five models produced equivalent estimates of the cumulative flux as a function of fetch over a wide range of atmospheric stability conditions despite the fact that some of them [*Legg and Raupach*, 1982; *Leclerc et al.*, 1988] do not satisfy the well-mixed criterion. The simulations were not overly sensitive to the specification of the drift correction term. In contrast to a previous study by *Baldocchi* [1992], the particle reflection scheme proposed by *Leclerc et al.* [1988] is comparable to the other drift correction terms for scalar flux estimation.

2. An imaginary "equivalent" uniform terrain subjected to identical surface roughness and turbulence statistics (i.e., mean, variance, and covariance) of velocities and scalars as those for the nonuniform terrain can be used for operational estimation of the latent heat flux profile. Good agreement between the eddy correlation measured and Lagrangian model predicted latent heat flux was demonstrated.

3. The usefulness of applying the Lagrangian model in estimating the latent heat flux profile, fetch requirement, and source weight function was tested by a "forward" and a "backward" approach. While *Leclerc and Thurtell* [1990] demonstrated the potentials of the Lagrangian method to estimate

the source weight function through their numerical simulations, the good agreement with field measurements and analytical solution reported here further confirms the ability of the Lagrangian models to predict the source weight function (i.e., the upwind source area) and fetch distances for water vapor.

4. The usefulness of the Lagrangian models to estimate latent heat fluxes under advective conditions for small roughness surfaces was evaluated with direct eddy correlation measurements. For unstable conditions the Lagrangian model adequately determines the fetch requirements, and the LE predictions are in good agreement with eddy correlation measurements. For near neutral conditions the limited internal boundary layer height (<1.96 m) posed difficulties in properly evaluating the Lagrangian model.

Appendix A

Instead of using the widely accepted Businger-Dyer stability correction functions for ϕ_h ($\phi_h = [1 - 16(z - d)/L]^{-1/2}$) in unstable conditions, we considered the function ($\phi_h = A_1[-(z - d)/L]^{-1/3}$) proposed by *Kader and Yaglom* [1990] and *Parlange and Katul* [1995] in the unstable conditions. Here A_1 (≈ 0.32) is a constant, and $\phi_h = 0.96$ in the stability range, $-(z - d)/L < 0.04$. In order to construct a continuous ϕ_h function for all the stability ranges, a constant A_2 is inserted to the function so that

$$\phi_h = A_1[A_2 - (z - d)/L]^{-1/3} \qquad (z - d)/L < 0 \tag{A1}$$

where $A_2 = (A_1/0.96)^3 = 0.037$. Some investigators estimate ϕ as ϕ_m rather than as ϕ_h in (12b). However, according to *Ley and Thomson* [1983], using ϕ_h is more accurate than using ϕ_m.

Appendix B

This appendix presents the analytical relationship between WTK and *Thomson*'s [1987] model and between WTK and WTK″.

Relationship Between the WTK and Thomson's Models

The WTK [*Wilson et al.*, 1981b] model suggests that for an inhomogeneous flow, the vertical velocity of a particle can be expressed as

$$q_{i+1} = \alpha q_i + \beta \eta_{i+1} \tag{B1}$$

$$w_i = \sigma_w(z_i) q_i + t_L(z_i)\sigma_w(z_i) \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_i} \tag{B2}$$

By combining (B1) and (B2), (B2) can be rewritten as

$$\frac{w_{i+1}}{\sigma_w(z_{i+1})} = q_{i+1} + t_L(z_{i+1}) \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_{i+1}} \rightarrow \frac{w_{i+1}}{\sigma_w(z_{i+1})}$$

$$= \alpha q_i + \beta \eta_{i+1} + t_L(z_{i+1}) \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_{i+1}} \rightarrow \frac{w_{i+1}}{\sigma_w(z_{i+1})}$$

$$= \alpha \left(\frac{w_i}{\sigma_w(z_i)} - t_L(z_i) \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_i} \right) + \beta \eta_{i+1} + t_L(z_{i+1}) \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_{i+1}} \tag{B3}$$

Now $\sigma_w(z_{i+1}) \approx \sigma_w(z_i) + \Delta t w_i \, \partial \sigma_w/\partial z|_{z=z_i}$ and $\Delta t \approx \gamma t_L$ (recall $\gamma = 1 - \alpha$), and by neglecting terms of order Δt^2 and assuming that $\partial \sigma_w/\partial z$ changes slightly with height, we then have

$$w_{i+1} = \left(\alpha \frac{w_i}{\sigma_w(z_i)} + \gamma t_L(z_{i+1}) \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_{i+1}} \right) \cdot \left(\sigma_w(z_i) + w_i \Delta t \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_i} \right) + \beta \eta_{i+1} \sigma_w(z_{i+1}) \rightarrow w_{i+1}$$

$$= \alpha w_i + \alpha \frac{w_i^2}{\sigma_w(z_i)} \Delta t \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_i} + \gamma t_L(z_{i+1}) \sigma_w(z_i) \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_{i+1}} + \beta \eta_{i+1} \sigma_w(z_{i+1})$$

Now for small $\Delta t/t_L$, $\sigma_w(z_{i+1}) \approx \alpha \sigma_w(z_i)$; we then have

$$w_{i+1} = \alpha w_i + \gamma t_L(z_{i+1}) \sigma_w(z_{i+1}) \cdot \left(\frac{w_i^2}{\sigma_w^2(z_i)} + 1 \right) \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_{i+1}} + \beta \eta_{i+1} \sigma_w(z_{i+1})$$

Hence the WTK model is equivalent to Thomson's model under these assumptions.

Relationship Between the WTK and WTK″ Models

A detailed derivation from the WTK model to the WTK″ model is shown. First, start with (B3):

$$\frac{w_{i+1}}{\sigma_w(z_{i+1})} = \alpha \left(\frac{w_i}{\sigma_w(z_i)} - t_L(z_i) \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_i} \right) + \beta \eta_{i+1} + t_L(z_{i+1}) \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_{i+1}} \rightarrow \frac{w_{i+1}}{\sigma_w(z_{i+1})}$$

$$= \alpha \frac{w_i}{\sigma_w(z_i)} + \beta \eta_{i+1} + \gamma t_L(z_{i+1}) \left.\frac{\partial \sigma_w}{\partial z}\right|_{z=z_{i+1}}$$

Now, by defining $w_{i+1}/[\sigma_w(z_{i+1})] = S_{i+1}$, we get the WTK″ model.

Appendix C

According to the test carried out by *Baldocchi* [1992] on *Leclerc et al.*'s [1988] reflection scheme, this model does not prevent the artificial particle accumulation near the ground surface. *Baldocchi* [1992] reported that near the ground surface, the turbulence is not strong enough resulting in a small travel distance of the particles and, consequently, the reflection ratio, p ($= \sigma_w(z_{i+1})/\sigma_w(z_i)$), is always near unity. Since the uniformly distributed random variables used to determine whether the reflection would occur are rarely greater than unity, the reflection, according to *Baldocchi* [1992], seldom occurs, and thus particles will accumulate. The following calculation shows that the particle reflection probability near the ground surface is about 21%, so it may be high enough to prevent particle accumulation in a low-turbulence region.

Consider a random number generator which has a uniform distribution between 0 and 1. The probability density function,

$f(x)$, of the random variable, x, is unity. The mean and variance of these random variables are [*Therrien*, 1992, chap. 2]

Mean

$$\bar{x} = E[x] = \int_0^1 xf(x)\, dx = 0.5$$

Variance

$$\sigma_x^2 = E[(x - \bar{x})^2] = E[x^2] - (E[x])^2$$

$$= \int_0^1 x^2 f(x)\, dx - (0.5)^2 = 1/12$$

To construct a uniformly distributed random draw with zero mean and unit variance, the random variable, x, has to be transformed by subtracting the mean ($\bar{x}$) and then dividing by the standard deviation (σ_x). This new random draw, x', is uniformly distributed between -1.73 and $+1.73$. Hence the probability of x' being greater than 1 is about 21%, which is not small.

However, this reflection probability analysis does not imply that the LTK model meets the well-mixed criterion [*Thomson*, 1987]. We have tested the well-mixed criterion with LTK and found that this model does not satisfy the criterion.

Appendix D

Before we started the two experiments at the Duke grass site, we compared the vertical velocity statistics measured by the Gill three-dimensional sonic anemometer and the Campbell CA27 one-dimensional sonic anemometer. The two anemometers were set next to each other (separation distance was 50 cm) at 2.6 and 1.54 m for the first and second experiments, respectively. Figure 10 shows a typical w time series comparison between the two anemometers (the mean of w measured from the three-dimensional anemometer was shifted by 6 m/s). Notice that the two measurements in Figure 10 are very compatible in terms of large excursions. Figure 11 compares the measured σ_w from the two anemometers and demonstrates that the one- and three-dimensional anemometers agree well for a wide range of atmospheric conditions ($+0.2 > z/L > -40$).

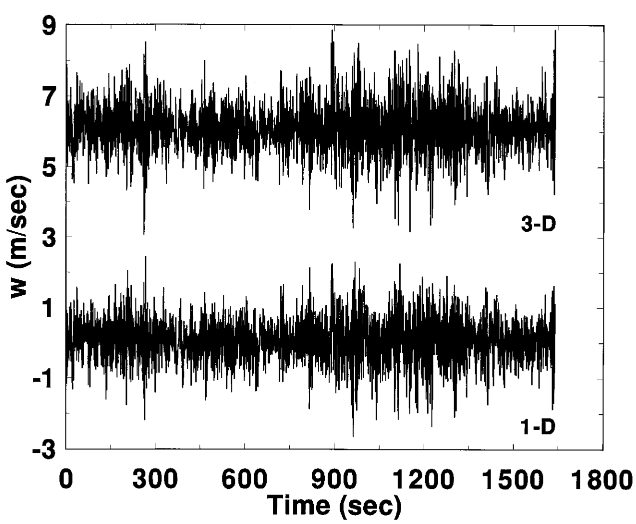

Figure 10. Comparison of vertical velocity (w) time series between the three-dimensional and one-dimensional sonic anemometer. The mean w measured by the three-dimensional anemometer was shifted by 6 m/s to permit comparison.

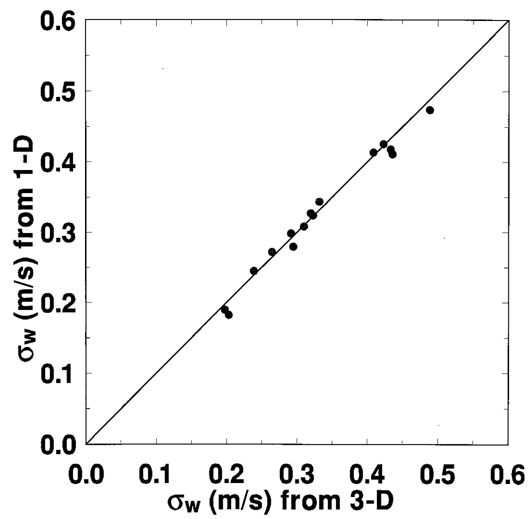

Figure 11. Comparison of measured σ_w between the three-dimensional and one-dimensional sonic anemometer. The 1:1 line is also shown.

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References

Angell, J. K., Lagrangian-Eulerian time-scale relationship estimated from constant balloon flights past a tall tower, *Adv. Geophys.*, *18A*, 419–431, 1974.

Baldocchi, D., A Lagrangian random-walk model for simulating water vapor, CO_2 and sensible heat flux densities and scalar profiles over and within a soybean canopy, *Boundary Layer Meteorol.*, *61*, 113–144, 1992.

Brutsaert, W., *Evaporation into the Atmosphere: Theory, History, and Applications*, 299 pp., D. Reidel, Norwell, Mass., 1982.

Calder, K. L., Some recent British work on the problem of diffusion in the lower atmosphere, *Proc. U.S. Tech. Conf. Air Poll.*, pp. 787–792, Mcgraw-Hill, New York, 1952.

Corrsin, S., Progress report on some turbulent diffusion research, *Adv. Geophys.*, *6*, 161–164, 1959.

Corrsin, S., Limitations of gradient transport models in random walks and in turbulence, *Adv. Geophys.*, *18A*, 25–60, 1974.

Csanady, G. T., *Turbulent Diffusion in the Environment*, 250 pp., D. Reidel, Norwell, Mass., 1973.

de Bruin, H. A. R., N. J. Bink, and L. J. M. Kroon, Fluxes in the surface layer under advective conditions, in *Land Surface Evaporation: Measurement and Parameterization*, edited by T. J. Schmugge and J. C. Andre, pp. 157–170, Springer-Verlag, New York, 1991.

Denmead, O. T., and E. F. Bradley, Flux-gradient relationships in a forest canopy, in *The Forest-Atmosphere Interaction*, edited by B. A. Hutchison and B. B. Hicks, pp. 421–442, D. Reidel, Norwell, Mass., 1985.

Dolman, A. J., and J. S. Wallace, Lagrangian and K-theory approaches in modeling evaporation from sparse canopies, *Q. J. R. Meteorol. Soc.*, *117*, 1325–1340, 1991.

Durbin, P. A., Stochastic differential equations and turbulent dispersion, *NASA Ref. Publ. 1103*, 69 pp., 1983.

Fischer, H. B., E. List, R. Koh, J. Imberger, N. Brooks, *Mixing in Inland and Coastal Waters*, 483 pp., Academic, 1979.

Gardiner, C. W., *Handbook of Stochastic Methods for Physics, Chemistry, and the Natural Sciences*, 442 pp., Springer-Verlag, New York, 1990.

Gash, J. H. C., A note on estimating the effect of a limited fetch on micrometeorological evaporation measurements, *Boundary Layer Meteorol.*, *35*, 409–413, 1986.

Hall, C. D., The simulation of particle motion in the atmosphere by a numerical random-walk model, *Q. J. R. Meteorol. Soc.*, *101*, 235–244, 1975.

Horst, T. W., and W. G. N. Slinn, Estimates for pollution profiles above finite area sources, *Atmos. Environ.*, *18*, 1339–1346, 1984.

Horst, T. W., and J. C. Weil, Footprint estimation for scalar flux measurements in the atmospheric surface layer, *Boundary Layer Meteorol.*, *59*, 279–296, 1992.

Kader, B. A., and A. M. Yaglom, Mean fields and fluctuation moments in unstable stratified turbulent boundary layers, *J. Fluid Mech.*, *212*, 637–662, 1990.

Katul, G. G., A model for sensible heat flux probability density function for near-neutral and slightly stable atmospheric flows, *Boundary Layer Meteorol.*, *71*, 1–20, 1994.

Katul, G. G., and M. B. Parlange, An atmospheric stability Penman Brutsaert potential evaporation model, *Water Resour. Res.*, *28*, 121–126, 1992.

Katul, G. G., J. D. Albertson, C. R. Chu, M. B. Parlange, H. Stricker, and S. Tyler, Sensible and latent heat flux predictions using conditional sampling methods, *Water Resour. Res.*, *30*, 3053–3059, 1994a.

Katul, G. G., M. B. Parlange, and C. R. Chu, Intermittency, local isotropy, and non-Gaussian statistics in atmospheric surface layer turbulence, *Phys. Fluid*, *6*, 2480–2492, 1994b.

Katul, G. G., S. M. Goltz, C. I. Hsieh, Y. Cheng, F. Mowry, and J. Sigmon, Estimation of surface heat and momentum fluxes using the flux-variance method above uniform and non-uniform terrain, *Boundary Layer Meteorol.*, *74*, 237–260, 1995.

Kustas, W. P., J. H. Blanford, D. I. Stannard, C. S. T. Daughtry, W. D. Nichols, and M. A. Weltz, Local energy flux estimates for unstable conditions using variance data in semiarid rangelands, *Water Resour. Res.*, *30*, 1351–1361, 1994.

Leclerc, M. Y., Turbulence and turbulent diffusion inside and above vegetation, Ph.D. thesis, Univ. of Guelph, Guelph, Ontario, Canada, 1987.

Leclerc, M. Y., and G. W. Thurtell, Footprint prediction of scalar fluxes using a Markovian analysis, *Boundary Layer Meteorol.*, *52*, 247–258, 1990.

Leclerc, M. Y., G. W. Thurtell, and G. E. Kidd, Measurements and Langevin simulations of mean tracer concentration fields downwind from a circular line source inside an alfalfa canopy, *Boundary Layer Meteorol.*, *43*, 287–308, 1988.

Legg, B. J., Turbulent dispersion from an elevated line source: Markov chain simulations of concentration and flux profiles, *Q. J. R. Meteorol. Soc.*, *109*, 645–660, 1983.

Legg, B. J., and M. R. Raupach, Markov-chain simulation of particle dispersion in inhomogeneous flows: The mean drift velocity induced by gradient in Eulerian velocity variance, *Boundary Layer Meteorol.*, *24*, 3–13, 1982.

Ley, A. J., A random walk simulation of two-dimensional turbulent diffusion in the neutral surface layer, *Atmos. Environ.*, *16*, 2799–2808, 1982.

Ley, A. J., and D. J. Thomson, A random walk model of dispersion in the diabatic surface layer, *Q. J. R. Meteorol. Soc.*, *109*, 847–880, 1983.

Luhar, A., and R. Britter, A random walk model for dispersion in inhomogeneous turbulence in a convective boundary layer, *Atmos. Environ.*, *23*, 1911–1924, 1989.

Luhar, A. K., and K. S. Rao, Source footprint analysis for scalar fluxes measured in the flows over an inhomogeneous surface, in *Air Pollution Modeling and Its Application*, edited by S.-E. Gryning and M. M. Millan, pp. 315–323, Plenum, New York, 1994.

Monin, A. S., and A. M. Yaglom, *Statistical Hydromechanics*, Hauka, Moscow, 1965.

Monin, A. S., and A. M. Yaglom, *Statistical Fluid Mechanics*, 769 pp., MIT Press, Cambridge, Mass., 1971.

Panofsky, H., and J. Dutton, *Atmospheric Turbulence: Models and Methods for Engineering Applications*, 397 pp., John Wiley, 1984.

Parlange, M. B., and Katul, G. G., Watershed-scale shear stress from tethersonde wind profile measurements under near neutral and unstable atmospheric stability, *Water Resour. Res.*, *31*, 961–968, 1995.

Parlange, M. B., G. G. Katul, R. H. Cuenca, M. Levent Kavvas, D. R. Nielsen, and M. Mata, Physical basis for a time series model of soil water content, *Water Resour. Res.*, *28*, 2437–2446, 1992.

Raupach, M. R., Canopy transport processes, in *Flow and Transport in the Natural Environment*, edited by W. L. Steffen and O. T. Denmead, pp. 95–127, Springer-Verlag, New York, 1988.

Raupach, M. R., Applying Lagrangian fluid mechanics to infer scalar source distributions from concentration profiles in plant canopies, *Agric. For. Meteorol.*, *47*, 85–108, 1989a.

Raupach, M. R., A practical Lagrangian method for relating scalar concentrations to source distributions in vegetation canopies, *Q. J. R. Meteorol. Soc.*, *115*, 609–632, 1989b.

Reid, J. D., Markov chain simulations of vertical dispersion in the neutral surface layer for surface and elevated releases, *Boundary Layer Meteorol.*, *16*, 3–22, 1979.

Sawford, B. L., Lagrangian statistical simulation of concentration mean and fluctuation fields, *J. Clin. Appl. Meteorol.*, *24*, 1152–1166, 1985.

Sawford, B. L., Generalized random forcing in random-walk turbulent dispersion models, *Phys. Fluids*, *29*, 3582–3585, 1986.

Sawford, B. L., Recent developments in the Lagrangian stochastic theory of turbulent dispersion, *Boundary Layer Meteorol.*, *62*, 197–215, 1993.

Schmid, H. P., Source areas for scalars and scalar fluxes, *Boundary Layer Meteorol.*, *67*, 293–318, 1994.

Schmid, H. P., and T. R. Oke, Estimating the source area of a turbulent flux measurement over a patchy surface, paper presented at 8th Symposium on Turbulence and Diffusion, Am. Meteorol. Soc., San Diego, Calif., April 26–29, 1988.

Schuepp, P. H., M. Y. Leclerc, J. I. Macpherson, and R. L. Desjardins, Footprint prediction of scalar fluxes from analytical solutions of the diffusion equation, *Boundary Layer Meteorol.*, *50*, 355–373, 1990.

Taylor, G. I., Diffusion by continuous movements, *Proc. Land. Math. Soc.*, *Ser. 2*, *20*, 196–211, 1921.

Tennekes, H., and J. L. Lumley, *A First Course in Turbulence*, 300 pp., MIT Press, Cambridge, Mass., 1972.

Therrien, C. W., *Discrete Random Signals and Statistical Signal Processing*, 727 pp., Prentice-Hall, Englewood Cliffs, N. J., 1992.

Thomson, D. J., Criteria for the selection of stochastic models of particle trajectories in turbulent flows, *J. Fluid Mech.*, *180*, 529–556, 1987.

Wilson, J. D., Turbulence measurements in a corn canopy and numerical simulation of particle trajectories in inhomogeneous turbulence, Ph.D. thesis, Univ. of Guelph, Guelph, Ontario, Canada, 1980.

Wilson, J. D., and B. L. Sawford, Review of Lagrangian stochastic models for trajectories in the turbulent atmosphere, *Boundary Layer Meteorol.*, *78*, 191–210, 1996.

Wilson, J. D., and G. E. Swaters, The source area influencing a measurement in the planetary boundary: The "footprint" and the "distribution of contact distance," *Boundary Layer Meteorol.*, *55*, 25–46, 1991.

Wilson, J. D., G. W. Thurtell, and G. E. Kidd, Numerical simulation of particle trajectories in inhomogeneous turbulence, I, Systems with constant turbulent velocity scales, *Boundary Layer Meteorol.*, *21*, 295–313, 1981a.

Wilson, J. D., G. W. Thurtell, and G. E. Kidd, Numerical simulation of particle trajectories in inhomogeneous turbulence, II, Systems with variable turbulent velocity scales, *Boundary Layer Meteorol.*, *21*, 423–441, 1981b.

Wilson, J. D., B. J. Legg, and D. J. Thomson, Calculation of particle trajectories in the presence of a gradient in turbulent-velocity variance, *Boundary Layer Meteorol.*, *27*, 163–169, 1983.

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